Spiraling of a confined elastic sheet after buckling

Stéphanie Deboeuf[1], Suzie Protière[1], Eytan Katzav[2]

[1] Sorbonne Université, CNRS, UMR 7190, Institut Jean Le Rond d'Alembert, F-75005 Paris, France
[2] Racah Institute of Physics, The Hebrew University, Jerusalem 9190401, Israel
stephanie.deboeuf@cnrs.fr

Plant leaves in buds, insect wings in cocoons or DNA in viral capsids are examples in nature of tightly packed flexible structures. One might wonder whether mechanics and physics of flexible sheets can help to understand geological folds [1]. However, the general equations describing deformations of tightly packed rods or sheets are challenging to solve. Here, we apply experimental and theoretical approaches to model the compression of a confined elastic sheet. Our experimental set-up consists of a sheet, whose ends are quasi-statically moved closer to each-other, while confined in the orthogonal direction by two walls separated by a given gap. Upon buckling and post-buckling of the sheet, as already observed by Roman and Pocheau [2], quasi-periodic wavy folded patterns appear. However, when pushed from the sides, as in the current configuration, a spontaneous instability occurs, leading to a single spiral pattern not reported before, but common to other confined configurations [3]. Experimentally, we measure simultaneously mechanical and geometrical properties of the system, using image analysis and force measurements. From a theoretical perspective, we compute predictions from the Euler-Bernoulli theory of elastic rods, both linearized and non-linear, inspired by the work of Chai [4]. A systematic comparison allows to investigate whether some regimes observed experimentally are well described by the linear theory and to identify the mechanisms that are necessary for the emergence of the spiral. Surprisingly, the appearance of the elastic spiral does not require friction, although the precise location of the transition is strongly affected when friction exists.

Références

1. J. Hall, *On the vertical position and convolutions of certain strata, and their relation with granite*, Earth & Environmental Sc. Trans. Royal Society of Edinburgh **7**, 79 (1815).
2. B. Roman, and A. Pocheau, *Buckling cascade of thin plates : Forms, constraints and similarity*, Europhys. Lett. **46**, 602 (1999).
3. L. Boué, M. Adda-Bedia, A. Boudaoud, D. Cassani, Y. Couder, A. Eddi, and M. Trejo, *Spiral patterns in the packing of flexible structures*, Phys. Rev. Lett. **97**, 166104 (2006).
4. H. Chai, *The post-buckling of a bi-laterally constrained column*, J. Mech. Phys. Solids **46**, 1155 (1998).