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# Instabilities and transition to turbulence in large aspect ratio wave attractors

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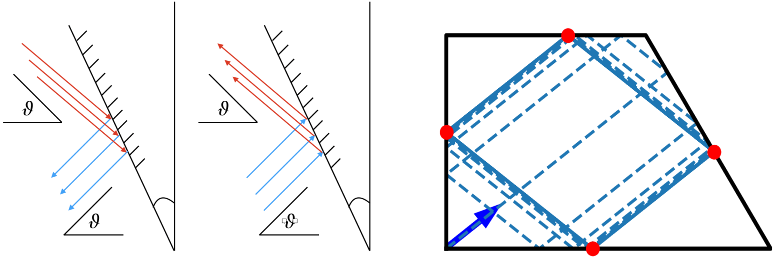
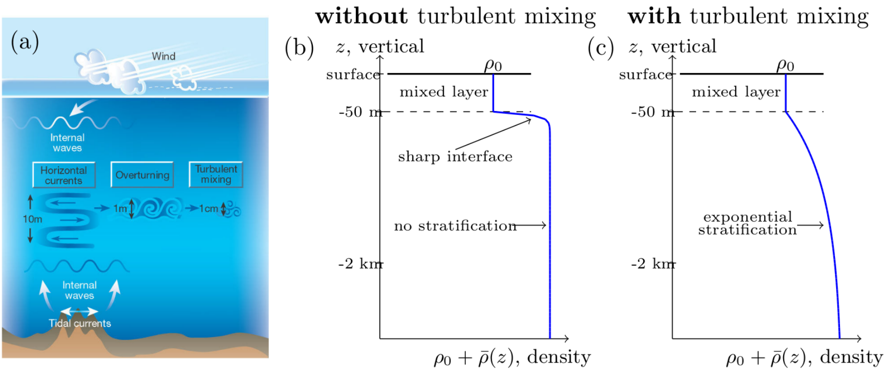
# Reflection from slopes: focusing



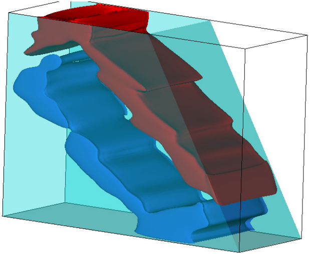

Schematic view of the reflection of an internal wave beam at a slope tilted with an angle $\alpha$. $c_g$ indicates the group velocity and $g$ the gravity.

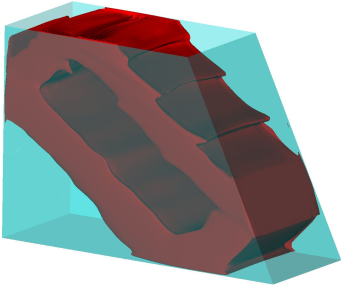
# Ocean abyssal stratification and internal waves



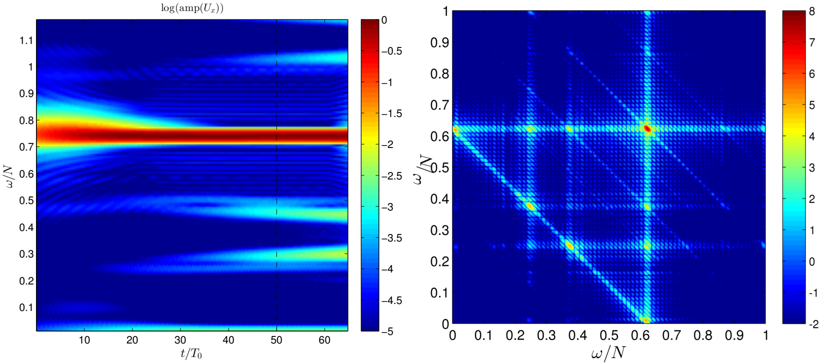
# Instabilities in 3D internal wave attractors (1,1).

Horizontal velocity in positive (red) and negative (blue) directions during instability development (LHS). Absolute value of instantaneous velocity vector during instability development (RHS).

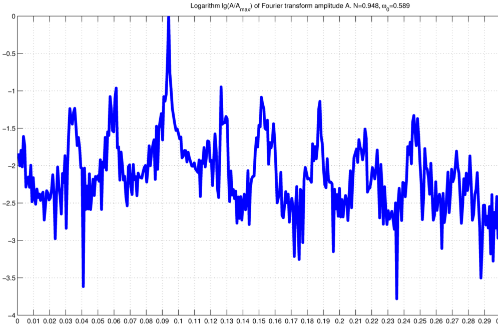


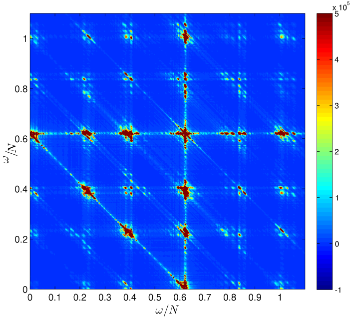


Time-frequency diagram and bispectrum demonstrate a perfect TRI.

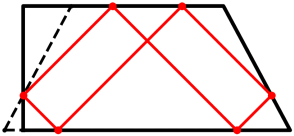


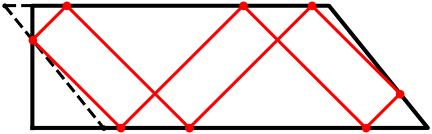
# 3D DNS Fourier transform and bispectrum, $t = 450..1770s$. Evidences of a TRI cascade

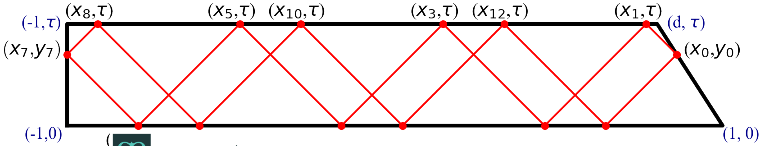


# Large aspect ratio domains








symmetry

*Article*

**On (n,1) Wave Attractors: Coordinates and Saturation Time**

**Ilias Sibgatullin [1,2,*], Alexandr Petrov [3], Xiulin Xu [4] and Leo Maas [5]**

$$x_R = n\tau - 1, \quad y_R = \frac{-n\tau^2 + 2\tau}{1-d};$$

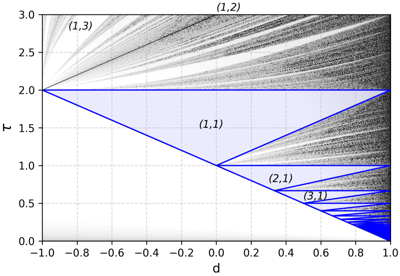
$$x_L = x_{n+1} = -1,$$

$$\text{for odd } n: \; y_L \equiv y_{n+1} = \frac{n\tau^2 - \tau d - \tau}{1-d},$$

$$\text{for even } n: \; y_L \equiv y_{n+1} = y_R;$$

$$x_k = \frac{-n\tau^2 + 2\tau}{1-d} - (k-n)\tau - 1, \; y_k = \frac{(-1)^{k+1}+1}{2}\tau, \; k = 1\ldots$$

$$x_k = \frac{n\tau^2 - 2\tau}{1-d} + (k-n-1)\tau - 1, \; y_k = \frac{(-1)^k + 1}{2}\tau, \; k = n+2\ldots$$

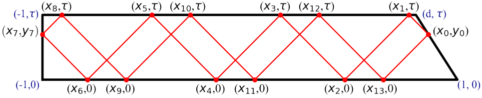



Areas of existence:

$$\tau > \frac{d+1}{n}; \quad \tau < \frac{2}{n}; \quad \tau > 1-d.$$

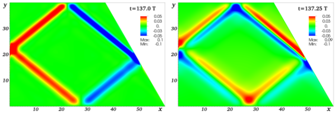
These inequalities define triangles in the $(d, \tau)$ plane with three vertices:

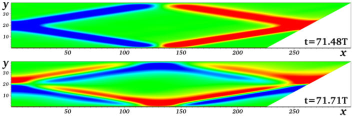
$$\left(1 - \frac{2}{n}, \frac{2}{n}\right), \left(\frac{n-1}{n+1}, \frac{2}{n+1}\right), \left(1, \frac{2}{n}\right).$$

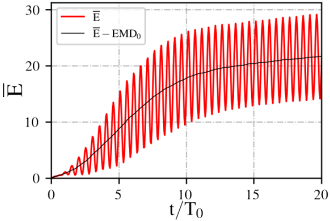


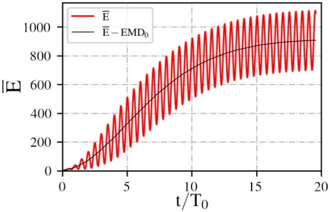

Expression for $x_R$ in (6) allows to give exact formulae for the Lyapunov exponents $\lambda$ in case of (n,1) attractors. Since the perimeter of the (n,1) attractor is equal to $2\sqrt{2}(x_R + 1)$ and the beams after reflections from the slope get closer by the focusing parameter $q = 1/\gamma$ [24]:

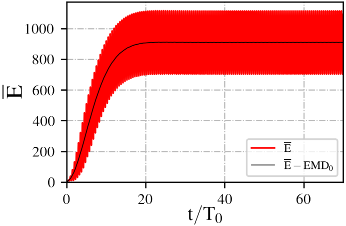
$$\lambda = \frac{1}{2\sqrt{2}(x_R+1)} \ln q = \frac{1}{2\sqrt{2}(x_R+1)} \ln \frac{1 - \tan(\alpha)}{1 + \tan(\alpha)} = \frac{1}{2\sqrt{2}n\tau} \ln \frac{\tau + d - 1}{\tau + 1 - d}. \quad (9)$$

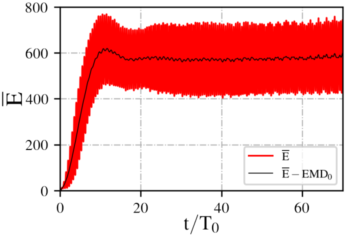

t=137.0 T
t=137.25 T
y
x


t=71.48T
t=71.71T
y
x


$\overline{E}$
$\overline{E} - EMD_0$
$t/T_0$


$\overline{E}$
$\overline{E} - EMD_0$
$t/T_0$


$\overline{E}$
$\overline{E} - EMD_0$
$t/T_0$


$\overline{E}$
$\overline{E} - EMD_0$
$t/T_0$

## Spectra and TFD, developement of waves at multiples of $f_0/2$

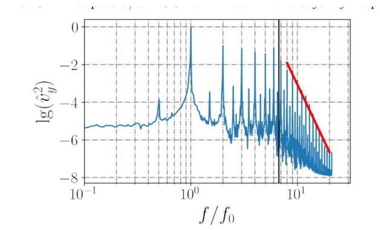



FIG. 9 Time spectra, $a = 0.12$ cm. Black line - buoyancy freq.

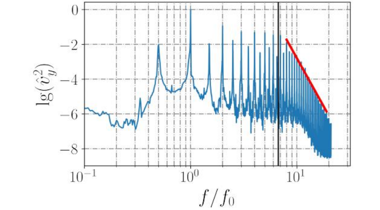



FIG. 10 Time spectra, $a = 0.14$ cm. Black line - buoyancy freq.

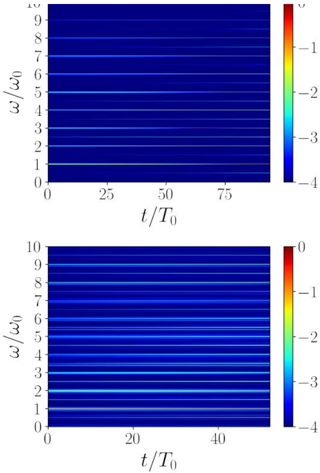
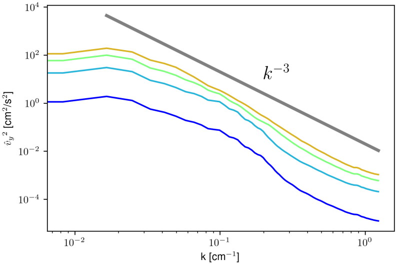
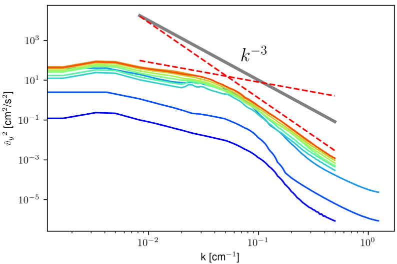
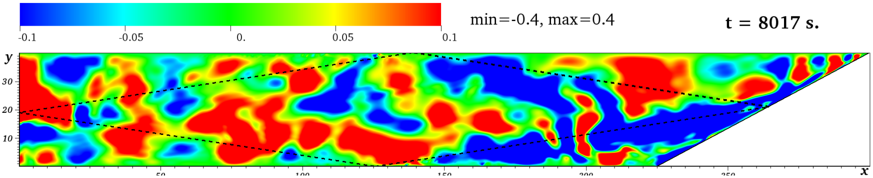


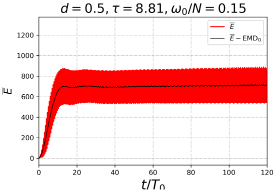

FIG. 11 Time-frequency diagrams, 0.00001, 0.01, 0.14, 0.22 cm


$10^4$
$10^2$
$10^0$
$10^{-2}$
$10^{-4}$
$\hat{v}_y{}^2$ [cm$^2$/s$^2$]
$k^{-3}$
$10^{-2}$
$10^{-1}$
$10^0$
k [cm$^{-1}$]

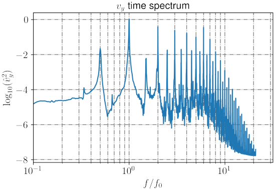

$10^3$
$10^1$
$10^{-1}$
$10^{-3}$
$10^{-5}$
$\hat{v}_y{}^2$ [cm$^2$/s$^2$]
$k^{-3}$
$10^{-2}$
$10^{-1}$
$10^0$
k [cm$^{-1}$]



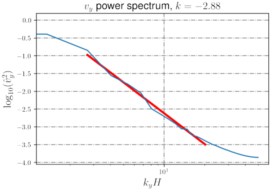
-0.1
-0.05
0.
0.05
0.1
min=-0.4, max=0.4
t = 8017 s.
y
30
20
10
50
100
150
200
250
x

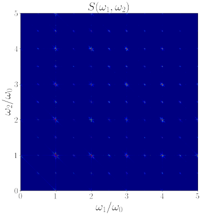
$d = 0.5, \tau = 8.81, \omega_0/N = 0.15$
$\overline{E}$
$\overline{E} - \mathrm{EMD}_0$
1200
1000
800
600
400
200
0
20
40
60
80
100
120
$t/T_0$

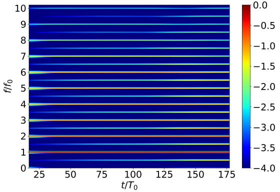
$v_y$ time spectrum
$\log_{10}(\hat{v}_y^2)$
$f/f_0$

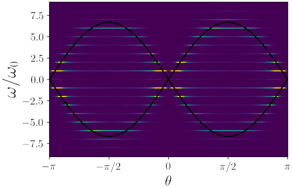
$v_y$ power spectrum, $k = -2.88$
$\log_{10}(\hat{v}_y^2)$
$k_y H$

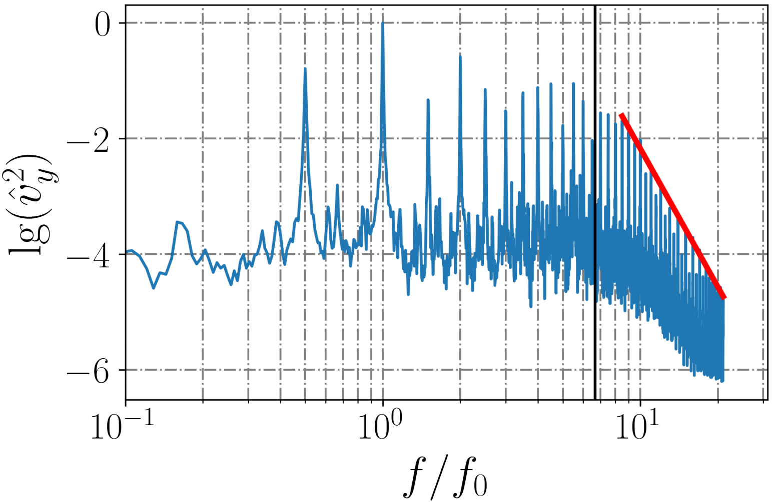
$S(\omega_1, \omega_2)$
$\omega_2/\omega_0$
$\omega_1/\omega_0$

$f/f_0$
$t/T_0$
25
50
75
100
125
150
175
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−4.0

$\omega/\omega_0$
$\theta$
$-\pi$
$-\pi/2$
0
$\pi/2$
$\pi$


0
−2
−4
−6
$\lg(\hat{v}_y^2)$
$10^{-1}$
$10^{0}$
$10^{1}$
$f\,/\,f_0$

# Conclusions

Triadic resonances are confirmed to be the primary cause of instability of internal wave attractors *for aspect ratio* $\approx 1$. In this case there is a strong evidence for a cascade of TRI with multi-peak discrete frequency spectrum.

The aspect ratio plays a crucial role in estimating the behavioral patterns during the nonlinear evolution of viscous internal wave regimes. Our findings enable the generalization of the triadic resonance instabilities (TRI) scenario.

In the frequency domain, if the interval between the forcing frequency and buoyancy frequency encompasses superharmonics of the forcing frequency $f_0$, then transition to turbulence as the forcing amplitude increases occurs as follows:

1. first, there is an energy pumping to the integer superharmonics between the forcing frequency and buoyancy frequency $f_0$;
2. next, there happens a hydrodynamical instability at multiples of half-frequency $f_0/2$,
3. and finally we have triadic resonances cascade in every half-$f_0$ interval