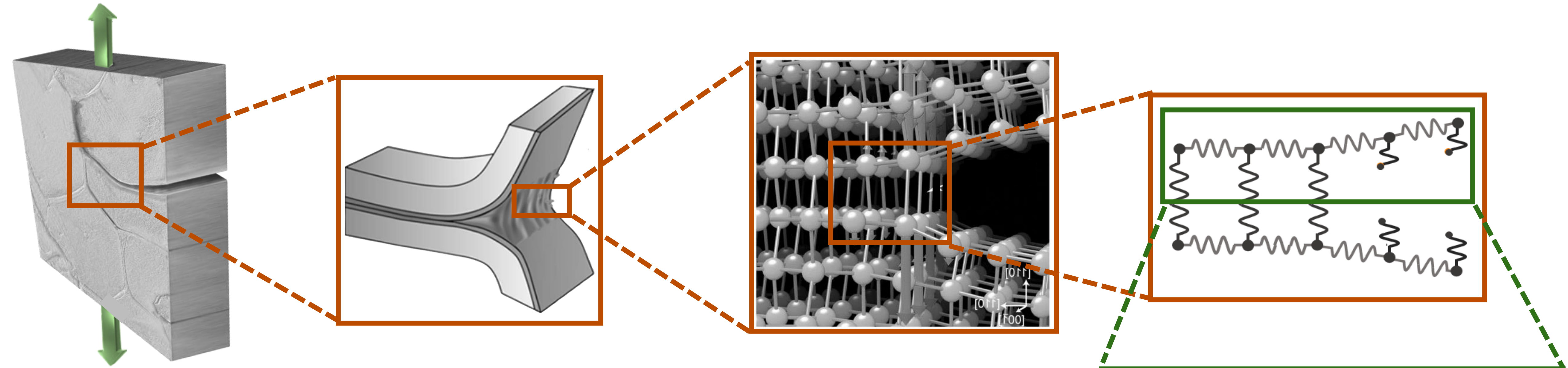
# Thermo-mechanical influence on fracture propagation: integrating temperature effects through equilibrium statistical mechanics

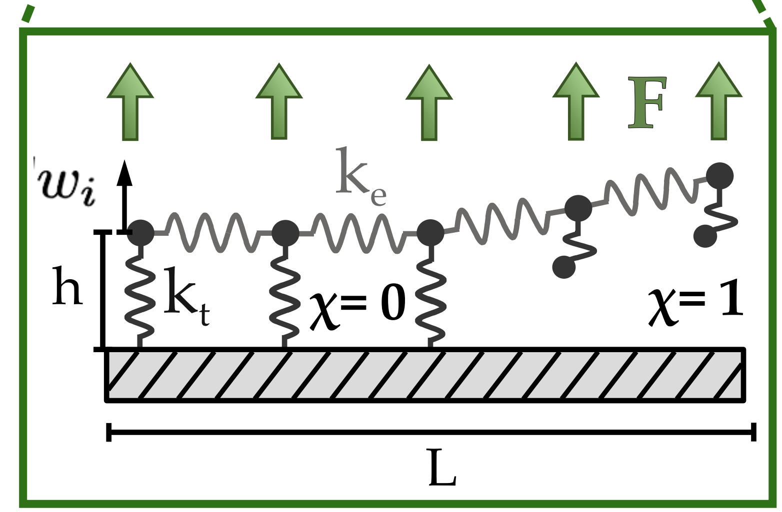


**Total mechanical energy**

$$g = \frac{1}{2}k_t \sum_{i=1}^{n} \left[(1-\chi_i)\, w_i^2 + \chi_i Y_m^2\right] + \frac{1}{2}k_e \sum_{i=1}^{n-1} (w_{i+1} - w_i)^2 - \sum_{i=1}^{n} F w_i$$

**Classical Griffith energy criterion for the mechanical system:**

$$\frac{\partial g_e}{\partial \xi} < 0 \quad \rightarrow \quad F > \frac{1}{1 + \frac{\xi}{\nu}\coth\left(\frac{1-\xi}{\nu}\right)}$$




Discrete model at the micro scale

27e Rencontre du non linéaire, Paris 19-20 March 2024

**C. Binetti**, G. Florio, N. M. Pugno, S. Giordano, G. Puglisi

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**Temperature effects**

Partition function in the Gibbs ensemble

$$\mathcal{Z}_G = \int_{\mathbf{R}^n} e^{-\frac{g}{k_B T}} dw_1 \dots dw_n$$

Gibbs free energy

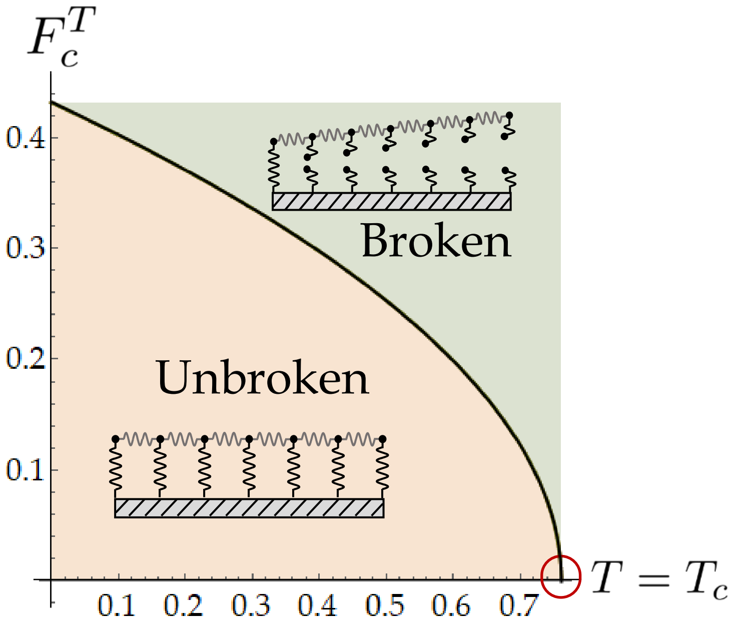
$$\mathcal{G} = H - TS = -k_B T \log \mathcal{Z}_G$$

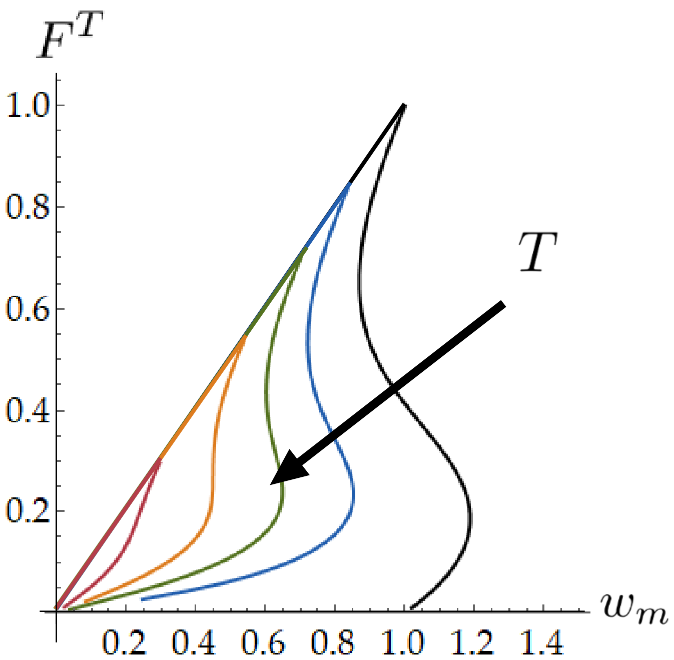
**Griffith energy criterion with thermal fluctuations**

$$\frac{\partial \mathcal{G}}{\partial \xi} < 0 \quad \rightarrow \quad F^T > \frac{\sqrt{1 - \frac{T}{T_c}}}{1 + \frac{\xi}{\nu} \coth\left(\frac{1-\xi}{\nu}\right)}$$

**Phase transition** (spontaneous rupture of the system) when $T = T_c$

$$T_c = \frac{\nu E L b Y_m^2}{h k_B \coth\left(\frac{1-\xi}{\nu}\right)}$$

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