Coalescence of viscous droplets under an elastic membrane

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The spreading of viscous fluids under an elastic membrane is an essential phenomenon underpinning many processes, both technical-industrial [1] and geological, such as certain laccolithic structures [2].

A particularly interesting case for this type of flow involves the study of the formation of two pockets of fluid between a rigid substrate on the one hand and an elastic membrane on the other. The drops generated will spread out until they eventually merge when their respective fronts meet. From this point onwards, a bridge of increasing height is formed.

This configuration is analogous to its capillary equivalent in the case of coalescence of free-surface drops. It was shown in [3] that the height of this bridge between the drops increases linearly with time, this process being described by a self-similar solution which has been made explicit in the same study.

We therefore propose to revisit this experiment in the case where we consider, not the surface tension, but the elasticity induced by the presence of the membrane.

We inject a fluid of viscosity μ and density ρ at a constant flow rate Q through a glass plate onto which a membrane of millimetric thickness d has been deposited. The evolution of the fluid's height profile $h(r,t)$ is therefore governed by the thin-film equation, where D and T represent the membrane's flexural modulus and tension respectively :

$$\frac{\partial h}{\partial t} = \frac{1}{12\mu r}\frac{\partial}{\partial r}\left(rh^3\left(\rho g\frac{\partial h}{\partial r} + D\frac{\partial}{\partial r}(\Delta_r^2 h) - T\frac{\partial}{\partial r}(\Delta_r h)\right)\right) + w(r, z=0, t) \tag{1}$$

Two drops are thus formed in the case of a double injection through two intrusions of diameter $e = 1$ mm and separated by a distance $L_{gap} = 3$ cm (cf. Fig. 1). We define the initial time t_0 of coalescence as the instant when the height of the bridge h_0 is non-zero.

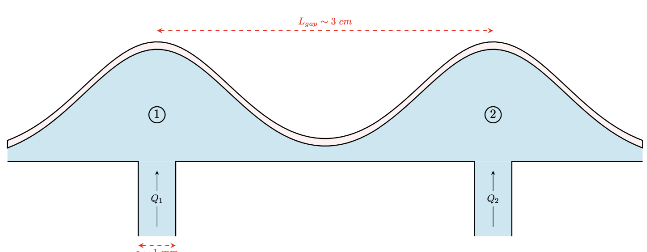

Figure 1. Simplified scheme representing the usual configuration of the study

We will show that the elasticity of the membrane is responsible for the relaxation of the drops as they merge by making explicite a power law confirming this dependancy. We will explain a simple geometric model that illustrates the self-similar behavior of the shape of the bridge joining the two blisters. We will try to confront the obtained results both experimentally and numerically.

Références

1. Fortunato, E. and Barquinha, P. and Martins, R., *Advanced Materials*, **24**, 2945-2986 (2012).
2. Castro, Jonathan M., C. Ian and Tuffen, Tobias S., *Nature Communications*, **7**, 13585 (2016).
3. Hernández-Sánchez, L. A. and Eddi, A. and Snoeijer, J. H., *Phys. Rev. Lett.*, **109**, 184502 (2012).