Wave packets that do not move at the group velocity

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The group velocity is an important unifying concept in wave phenomena. It yields the speed of wave packets and generally differs from that of the phase velocity of the underlying oscillations. Moreover, its validity carries over to the realm of weakly nonlinear wave packets through the universal nonlinear Schrodinger equation (NLS). However, this communication reports that in this general frame, the group velocity, and hence the NLS, ceases to describe the dynamics of soliton wave packets when the group and phase velocities are sufficiently close to one other.

We will demonstrate our result on the following general class of wave equations

$$\frac{\partial^2 E}{\partial x^2} + \int_{-\infty}^{\infty} \beta^2(\omega)\hat{E}(x,\omega)e^{-i\omega t}\mathrm{d}\omega = N[E,\epsilon]\,,$$

where

$$\hat{E}(x,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E(x,t)e^{i\omega t}\mathrm{d}t$$

is the Fourier transform of the scalar field $E(x,t)$ of interest and $N[E,\epsilon]$ is the nonlinearity, which tends to zero as $\epsilon \to 0$. The linear properties of the wave system is contained in the dispersion function $\beta(\omega)$ and the results are derived in a region of parameters such that the relative difference between the group and phase velocities is $O\left(\epsilon^{-1/2}\exp(-\pi/4\epsilon)\right)$. In that region, not only do solitonic wave packets depart from the group velocity, but worse than that : they do not even move at a constant speed. In this presentation, we will provide a physical argument for this behaviour and sketch the derivation based on exponential asymptotics [1]. The final result is that the motion of the wave packet is governed by a surprisingly simple and familiar equation.

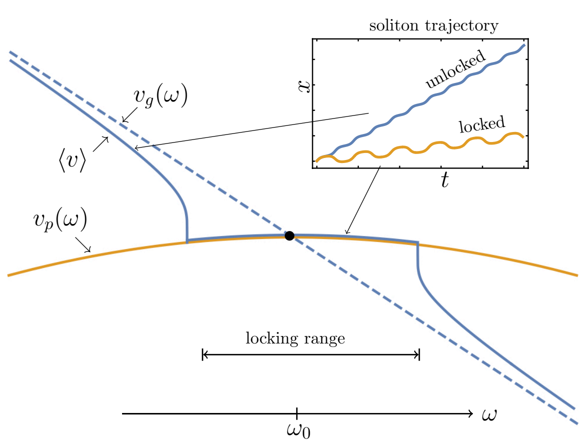

Figure 1. v_g : group velocity, v_p : phase velocity, $\langle v\rangle$: average soliton velocity. Inset : soliton trajectory.

Références

1. G. Kozyreff, *Phys. rev. E*, **107**, 014219 (2023).