# Modelling, Analysis, and Finite Element simulations of kinematically incompatible Föppl-von von Kármán plates.

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In the first part of the talk, I investigate the existence and regularity of solutions for kinematically incompatible Föppl-von Kármán (FvK) thin plates [1,2] when the plate boundaries is free of in-plane traction and the out-of-plane displacement is clamped. The kinematic incompatibility arises from the presence of Volterra wedge disclinations in the crystalline lattice and enters the mathematical formulation as a discrete distribution of Dirac delta measures. The mathematical model consists of two coupled, nonlinear, fourth-order elliptic partial differential equations.
The existence of solutions is rigorously established using the direct method in the calculus of variations, following a similar approach to Ciarlet's proof [3] for kinematically compatible plates (i.e. in plates whose crystalline lattice contains no defects). The regularity of the solutions is also demonstrated under additional assumptions regarding the regularity of the boundary of the plate and of the external load.
In the second part of the talk, I introduce a novel Interior Penalty $C^0$-Discontinuous Galerkin (IPDG) [4] Finite Element formulation to numerically solve the kinematically incompatible FvK plate problem. The IPDG formulation proposed extends the work presented in [5] by applying the IPDG methodology at the energy level rather than directly in its weak form. To conclude, I perform some numerical experiments by varying the two key nondimensional parameters of the problem within appropriate ranges.
A direct application of this study is in the continuum modelling of graphene sheets with wedge disclinations in the crystal lattice.

## References

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