Computing arc-lengths to portray chaos

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Chaotic dynamics is not a rare phenomenon in nature, manifesting in various physical or biological systems. Due to its prevalence, significant efforts over the past decades have been dedicated to develop numerical indicators that capture its signatures. In addition to spectral methods, a widely used category of chaos indicators consists of variational methods relying either on the tangent map for discrete dynamics and variational equations for continuous dynamics, such as Lyapunov exponents, fast Lyapunov indicators, mean-exponential growth of nearby orbits or the generalised alignement index to name but a few.

This contribution will present and review a recently introduced indicator that follows from Lagrangian descriptors (LDs) [1]. The index encapsulates the regularity of the length metric, computed over a finite time window, by estimating its unmixed second derivatives with respect to the initial condition. The index is refereed as $||\Delta \mathrm{LD}||$ [2]. We will discuss the performance of the proposed chaos indicator on the standard map and higher dimensional versions, demonstrating its effectiveness in distinguishing between regular and chaotic dynamics. For this, the required derivatives are estimated using two different approaches: finite differences (FD) methods computed on fine meshes of initial conditions and differential algebra (DA) serving as a form of automatic differentiation. Following [3], our numerical explorations show that FD methods might lead to a significant misclassification rate, reaching up to 20% in the mixed phase space regime when the phase space supports thin chaotic and resonant web. In contrast, the computation of the chaos indicator using DA proves to be highly reliable with a level of sensitivity almost identical to well-established first-order tangent map-based indicators. Moreover, this accuracy is maintained regardless of the specific characteristics of the phase space.

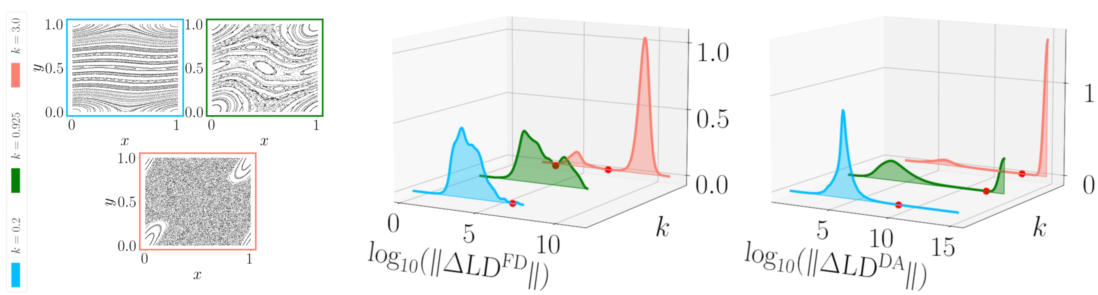

Figure 1. Distributions of $||\Delta \mathrm{LD}^{\mathrm{FD}}||$ and $||\Delta \mathrm{LD}^{\mathrm{DA}}||$ on fine initial condition meshes. The differential algebra framework yields sharp distributions contrarily to finite differences, thus improving trajectory classification. Results are shown for the standard map with three nonlinearity parameters for which their corresponding phase spaces are shown.

References

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