Shell Models on Recurrent Sequences

Lorenzo Manfredini[1], Özgür D. Gürcan[1]

Laboratoire de Physique des Plasmas, CNRS, Ecole Polytechnique, Palaiseau, France
lorenzo.manfredini@lpp.polytechnique.fr

Shell models are a particular class of toy models in tubulence research, whose dynamics successfully mimic some nontrivial aspects of the turbulent cascade, such as power-law spectra and intermittency.
The standard archetype of shell model is the GOY model, with a logarithmic spacing $g = 2$. Considering this as a 1D decimation of the wave-number domain, for example in the spirit of nested polyhedra [1] models or log-lattice models [2], the shell spacing imposed may be the golden-ratio, or other special numbers. However interpreting such models as decimations of a Fourier space grid is complicated, since the wave-numbers that are computed on such logarithmic grids do not fall on integer values.
In order to remedy this, we can use recurrent sequences. It is possible to construct models where shell wave-numbers are integers and asymptotically self-similar by constuction, hence enabling the connection between a regular grid and a logarithmic lattice.
In order to test this approach, we propose a new class of shell models [3], where the shell variables are defined on a recurrent sequence of integer wave-numbers such as the Fibonacci series, and other variations. Considering the simplest model, which involves only local interactions, the interaction coefficients can be generalized in such a way that the inviscid invariants, such as energy and helicity, can be conserved even though there is no exact self-similarity. It is shown (cf. Fig. 1) that these models basically have identical features with standard shell models.

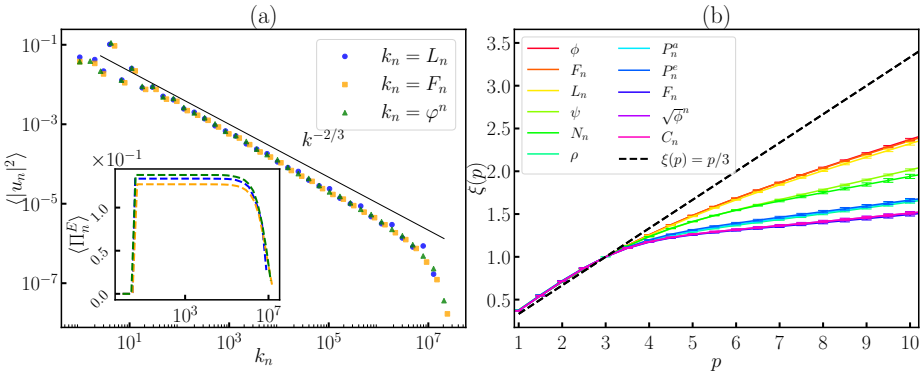

Figure 1. (a) Energy spectrum and flux for the shell model implemented on recurrent sequences and their corresponding self similar sequence, showing how the proposed model displays the same features of standard shell models without the recuirement of exact self-similarity. (b) Intermittency corrections for the various recurrent series considered, showing comparable level of intermittency for sequences corresponding to the same asymptotically self-similar intershell ratio. Smaller inter-shell ratios are associated with an increase of the intermittency corrections.

References

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