

# Déstabilisation par un processus d'advection dans un laser: défauts spectro-temporels et structures induites par bruit

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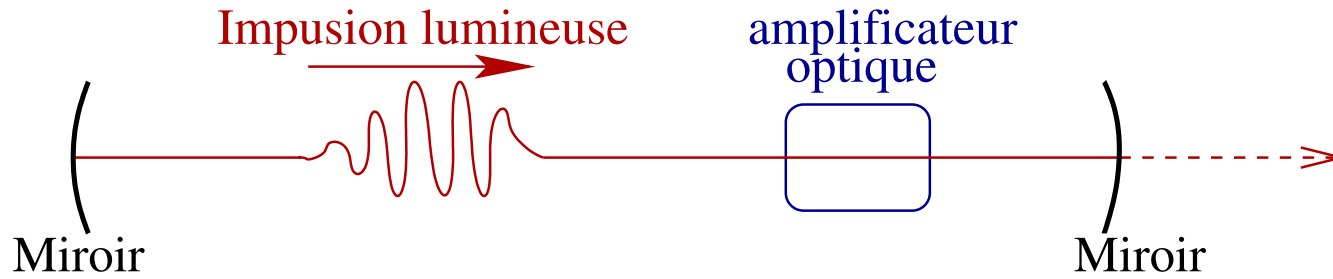
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*Université Paris-Sud, Orsay (France)*

M. Hosaka, A. Mochihashi, and M Katoh

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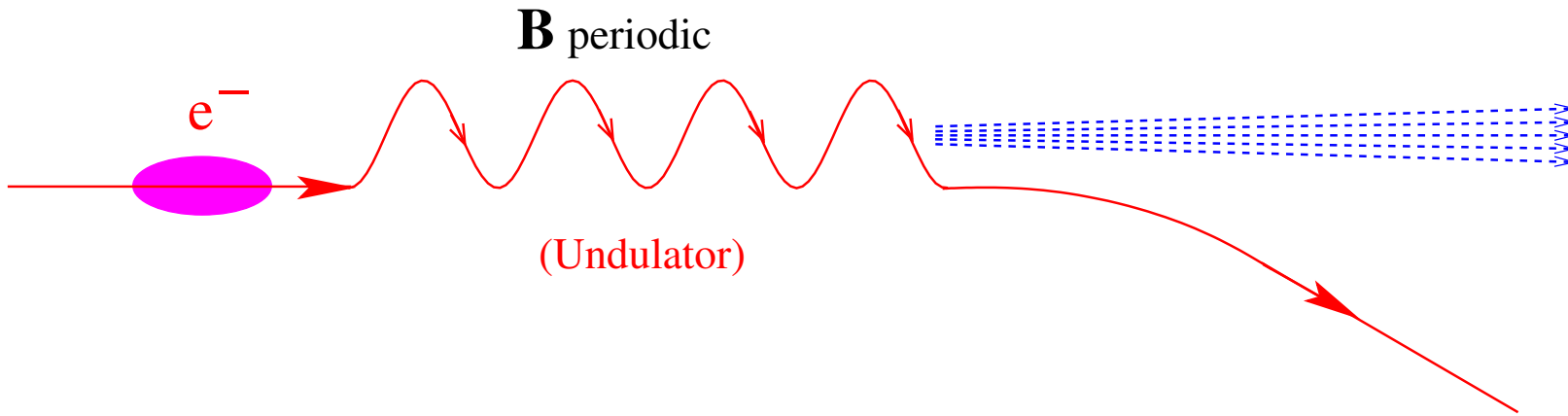
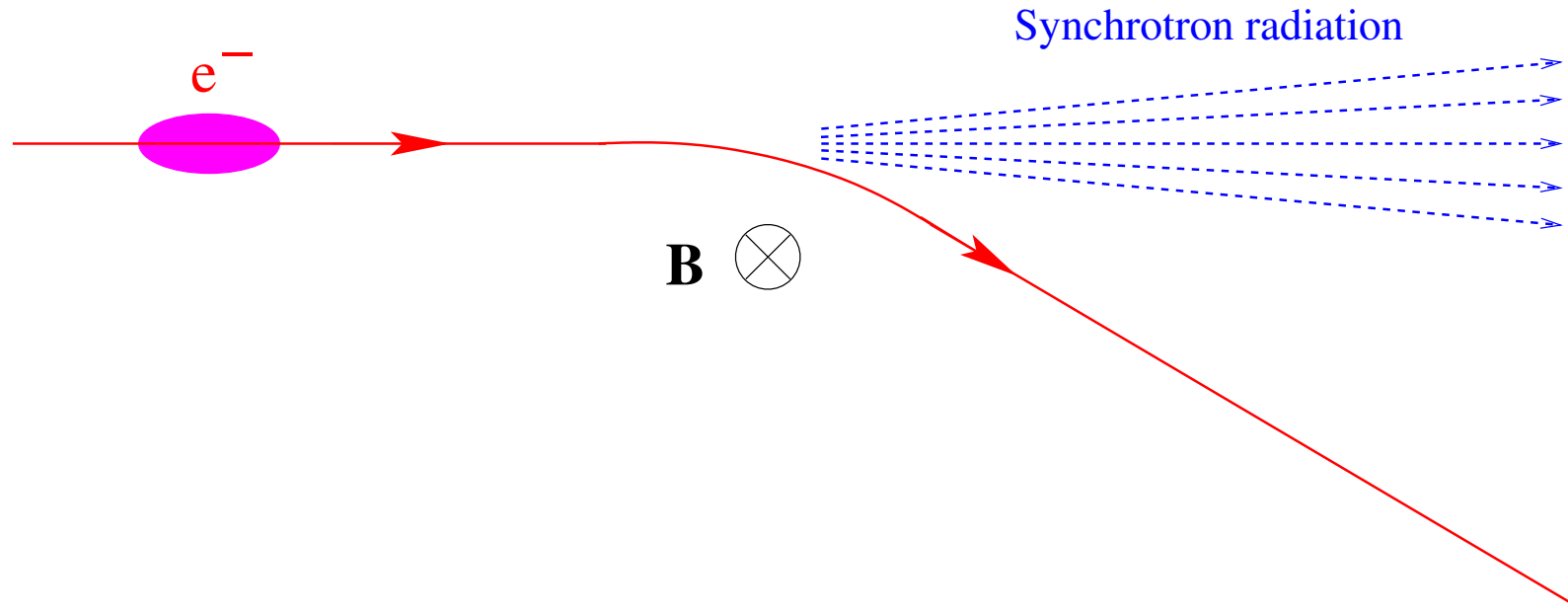
1. Le laser à électrons libres:

aspect physique  $\longrightarrow$  expérience et modèle  
 aspect "dynamique"  $\longrightarrow$  système du type "advection–diffusion" à 1d  
 + saturation globale

2. Résultats (quoi de neuf?)  $\longrightarrow$  dynamique spectro–temporelle  
 $\searrow$  saturation globale vs saturation locale

3. Quels phénomènes retrouve t–on dans des équations de Ginzburg–Landau élémentaires?





electromagnetic  
wave



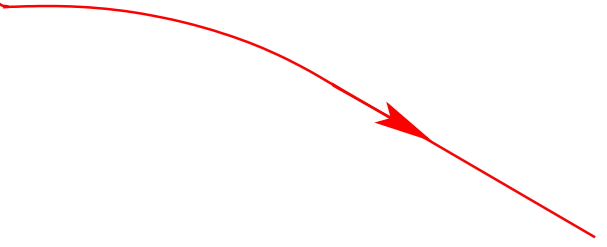
$e^-$

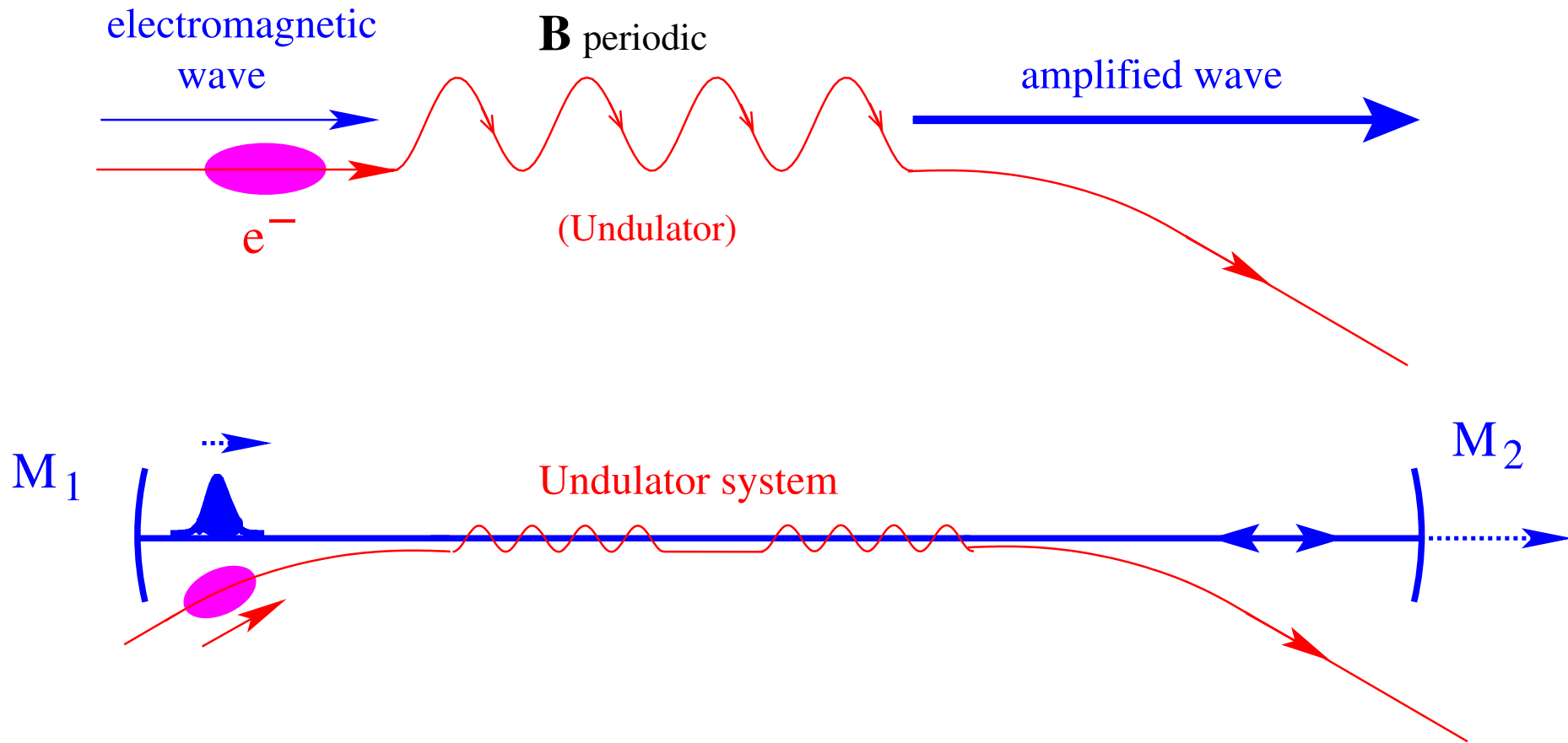


**B** periodic

(Undulator)

amplified wave



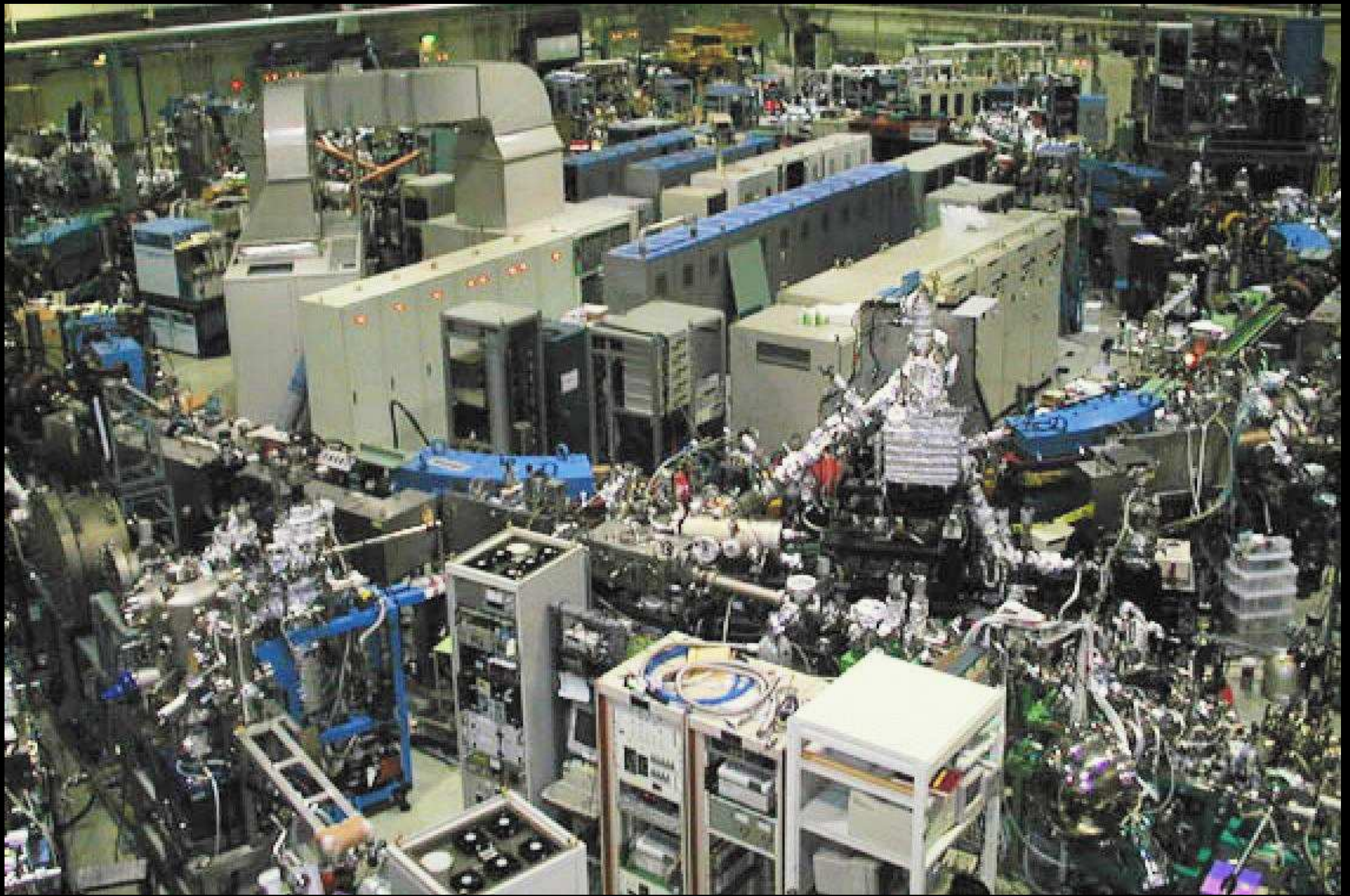


- Free–electron laser (FEL):
- tunable
  - far infrared to UV (and X...)

Super–ACO (LURE, Orsay, France) .....➔ UV (350 nm)

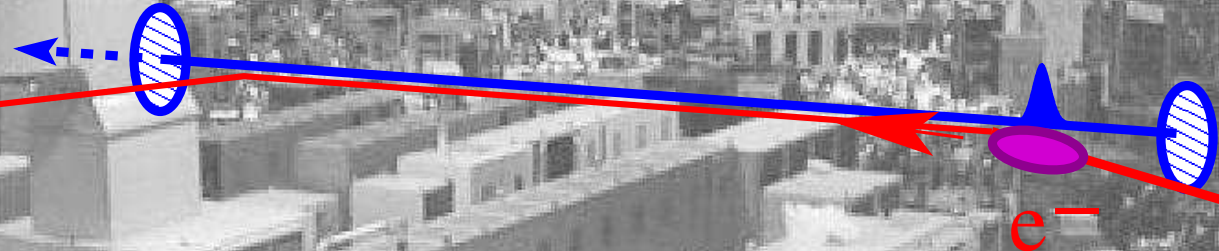
UVSOR (IMS, Okazaki, Japan) .....➔ UV (250 nm), visible (520 nm), etc

# UVSOR storage ring (IMS, Okazaki)



$M_2$

$M_1$



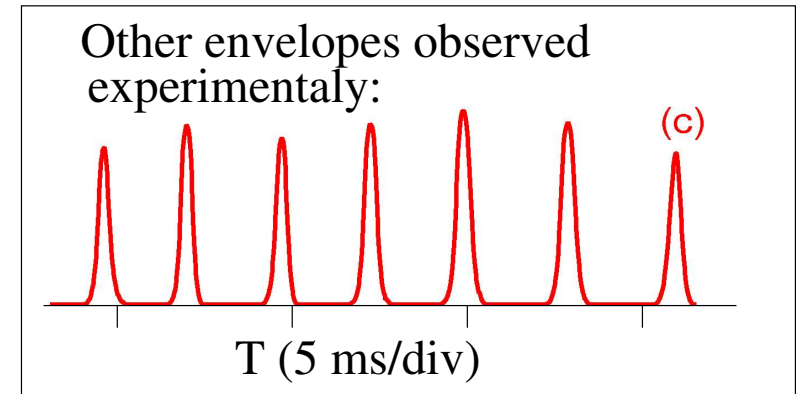
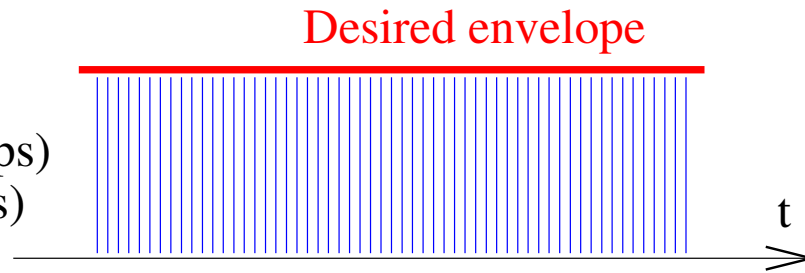
Energy: 800 MeV  
Lifetime: several hours  
Revolution frequency: 5.6 MHz  
Current: tens of mA



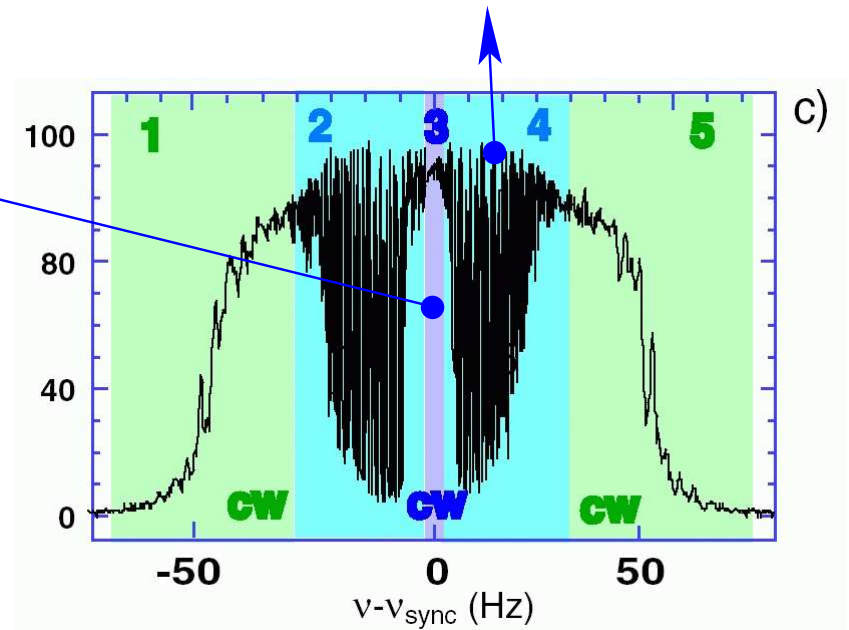
# Stability issues vs frequency mismatch

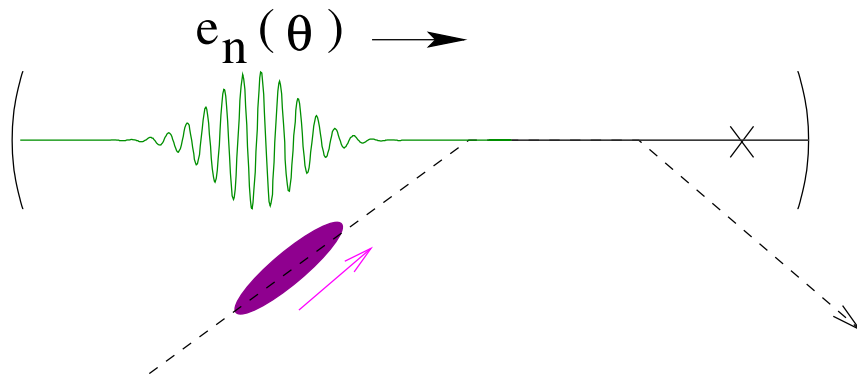
## 1) Envelope of the pulse train

laser pulses:  
duration=  $O(10\text{ps})$   
period= $O(100\text{ns})$

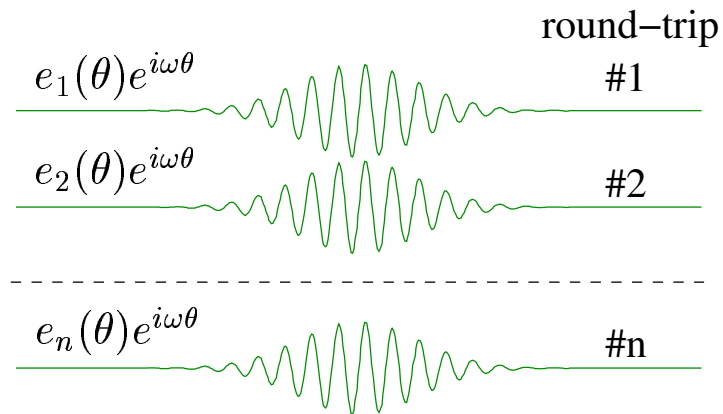


## 2) Internal structure of the picosecond pulses ? (this talk)





"space"  $\theta$

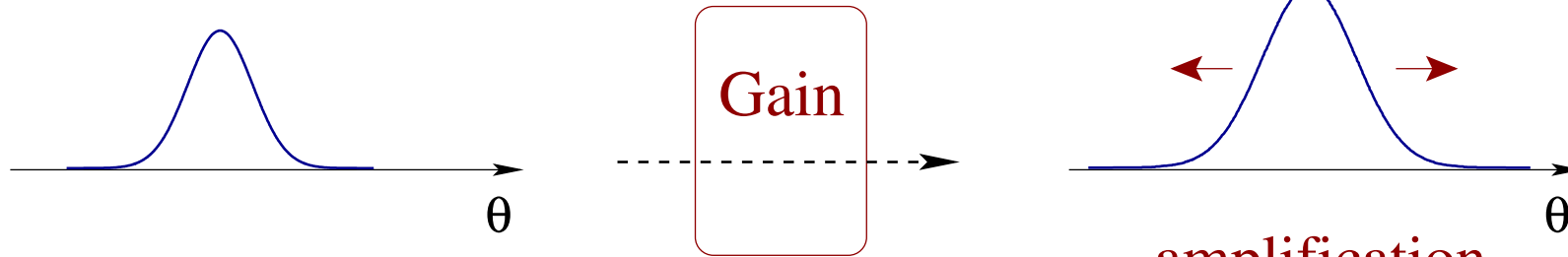


*discrete time*  
*(round-trip n)*

– at each round-trip:

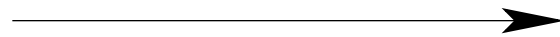
$$e_n(\theta) \xrightarrow{\text{loss, gain}} e_{n+1}(\theta)$$

Effet du gain ?



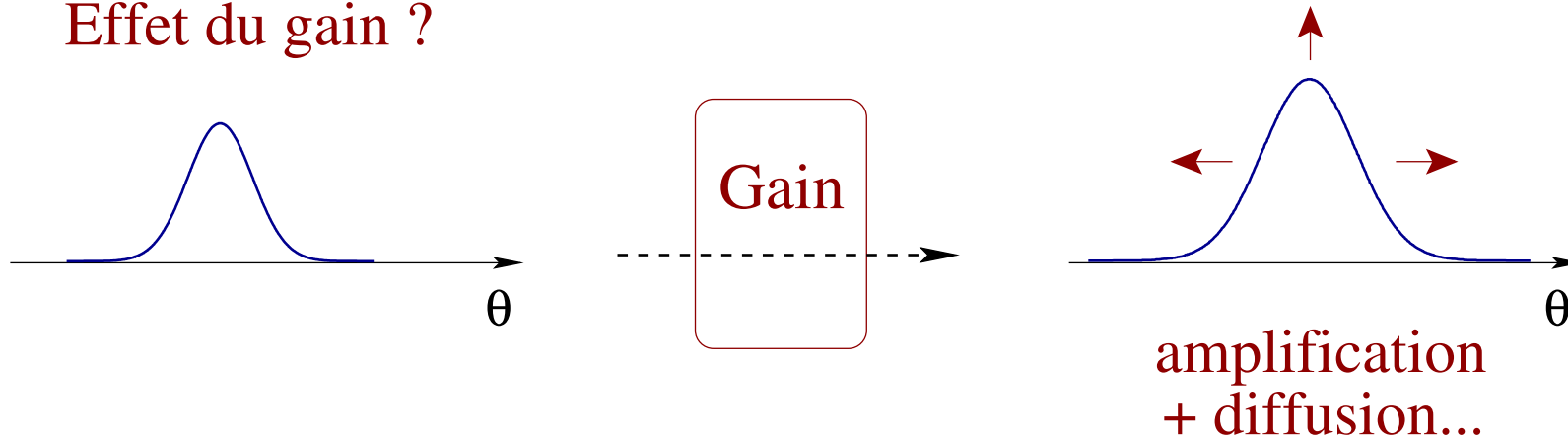
amplification  
+ diffusion...

$$e(\theta)$$

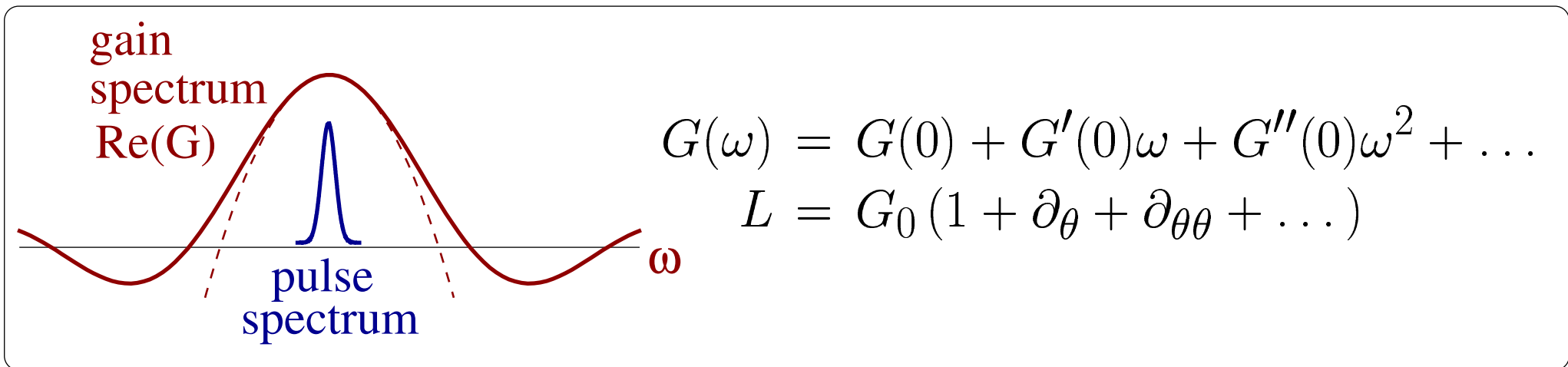


$$L \otimes e(\theta) = G_0 \times (e + e_{\theta\theta})$$

# Effet du gain ?



$$\begin{array}{ccc}
 e(\theta) & \xrightarrow{\hspace{10em}} & L \otimes e(\theta) = G_0 \times (e + e_{\theta\theta}) \\
 \text{TF} \updownarrow & & \updownarrow \\
 \tilde{e}(\omega) & \xrightarrow{\hspace{10em}} & G(\omega)\tilde{e}(\omega)
 \end{array}$$

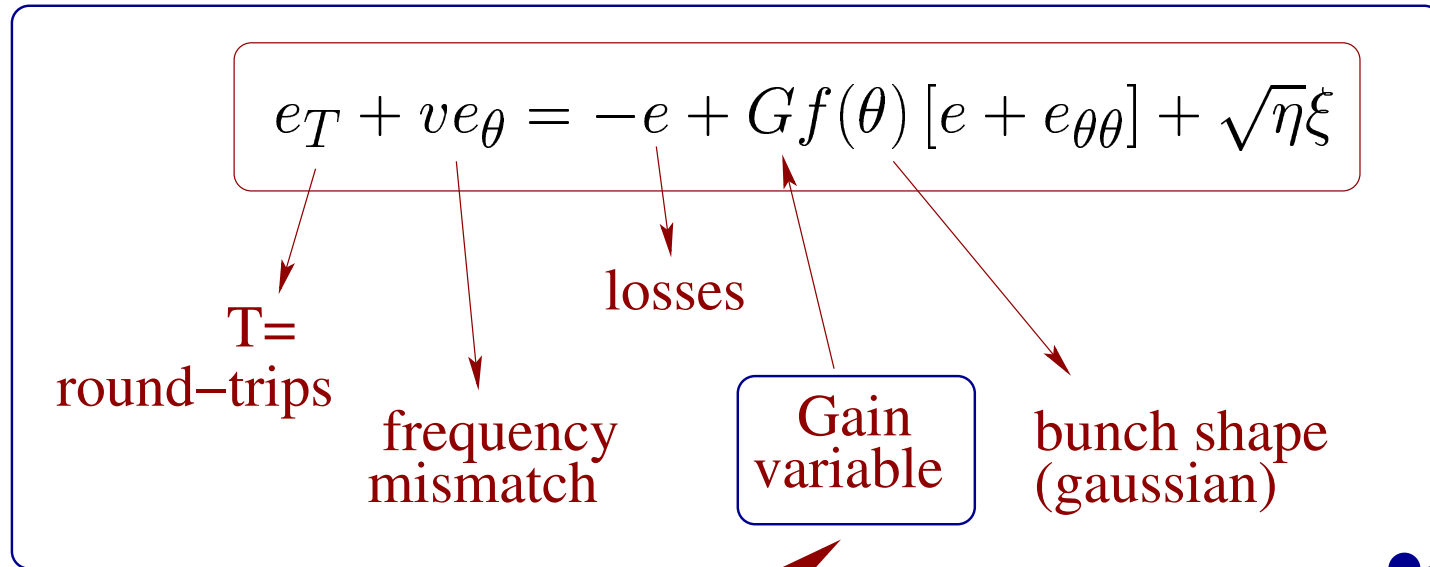


$$G(\omega) = G(0) + G'(0)\omega + G''(0)\omega^2 + \dots$$

$$L = G_0 (1 + \partial_\theta + \partial_{\theta\theta} + \dots)$$

Final step: continuous limit: Map  $\rightarrow$  PDE

Pulse  
shape  
 $e(\theta, T)$



Gain depends only  
on slow time  $T$

$G =$  decreasing function of temperature  $\sigma^2$

$$\frac{d\sigma^2(T)}{dT} = -\frac{1}{T_s} \left( 1 - \sigma^2 + \int_0^L |e(\theta, T)|^2 d\theta \right)$$

Gain saturation

depends on overall power

competition between

advection

diffusion

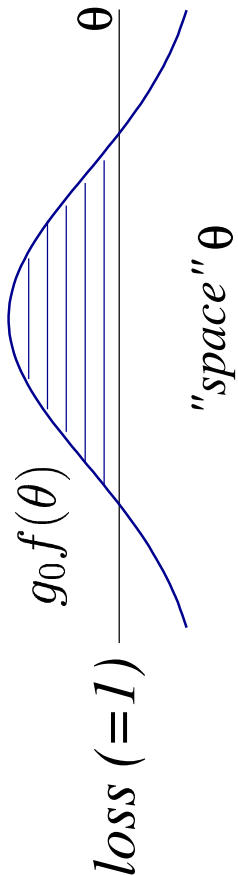
$$e_T + ve_\theta = -e + g(T)f(\theta)(e + e_{\theta\theta}) + \sqrt{\eta}\xi,$$

*v large:*  $v > v_{cv}$

transient growth  
 $e=0$  is globally stable  
 (convective instability)

*v not too large:*  $v < v_{cv}$

(absolute instability)

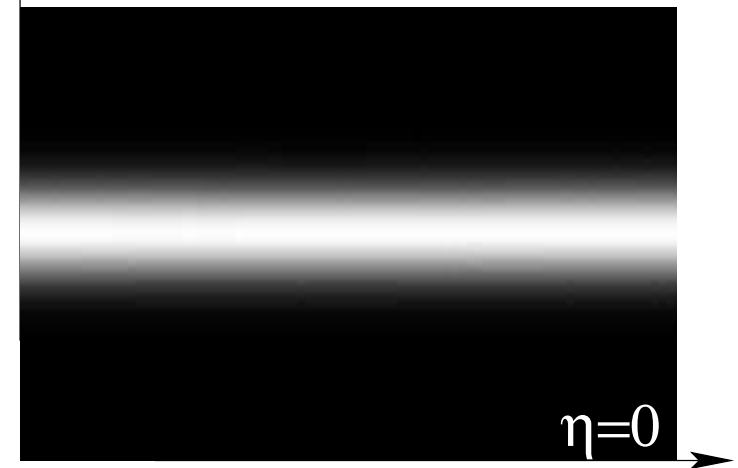


"space"  $\theta$



Time  $T$  (number of round-trips)

"space"  $\theta$



Time  $T$  (number of round-trips)

$$v_{cv} = 2\sqrt{g_0(g_0 - 1)}$$

Basic concepts: see eg Huerre and Monkewitz, *Ann. Fluid Mech.* 22, 473 (1990),

competition between

advection

diffusion

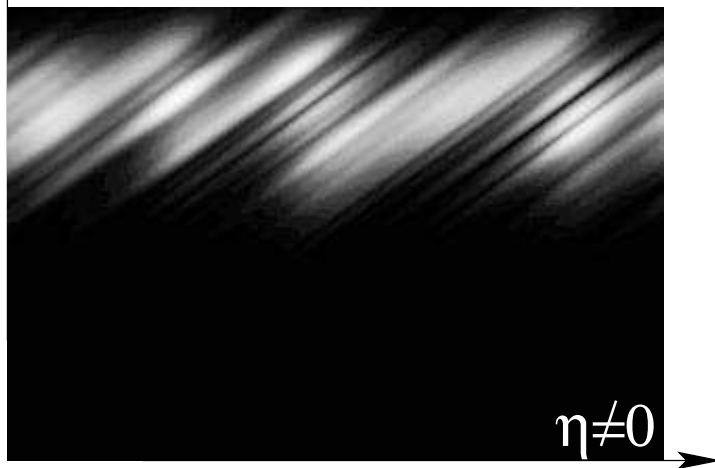
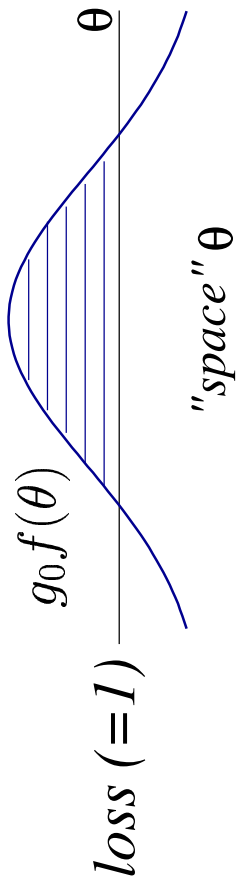
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$v$  large:  $v > v_{cv}$

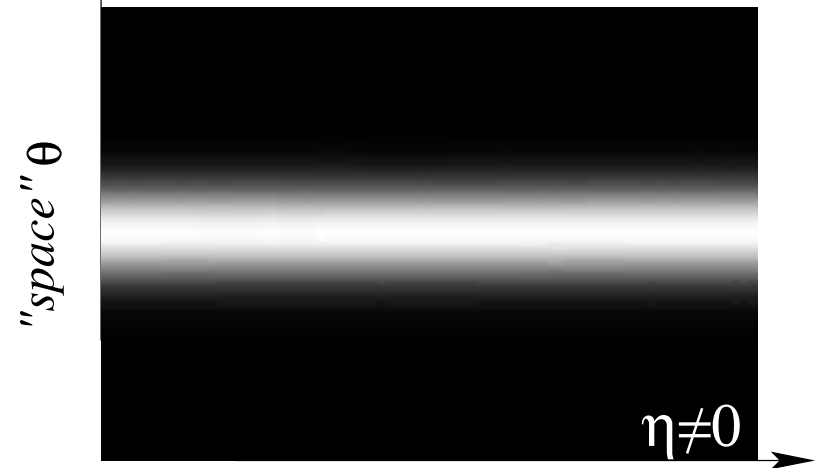
transient growth  
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$v$  not too large:  $v < v_{cv}$

(absolute instability)



Time  $T$  (number of round-trips)



Time  $T$  (number of round-trips)

Transient growth known in mode locked lasers:

*Kartner et al. PRL 82, 4428 (1999)*

*Morgner & Mitschke, PRE58, 187 (1999)*

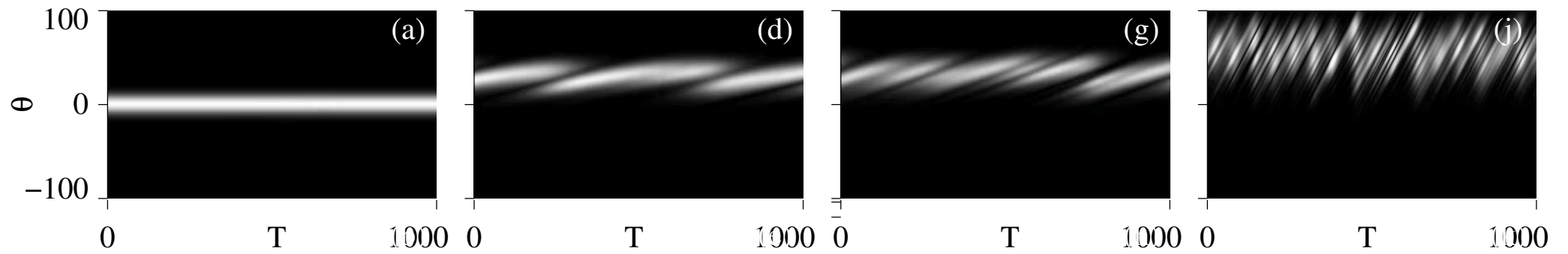
Basic concepts on cv instabilities: see eg *Sturrock, Phys. Rev. 5, 488 (1958)*

*Huerre and Monkewitz, Ann. Fluid Mech. 22, 473 (1990),*

*Fluid ex.(Hele-Shaw cell) PRL 82, 1442 (1999)*

*Cossu & Chomaz PRL 78, 4387 (1997)*

## Numerical results



Frequency  
mismatch:  $v=0$

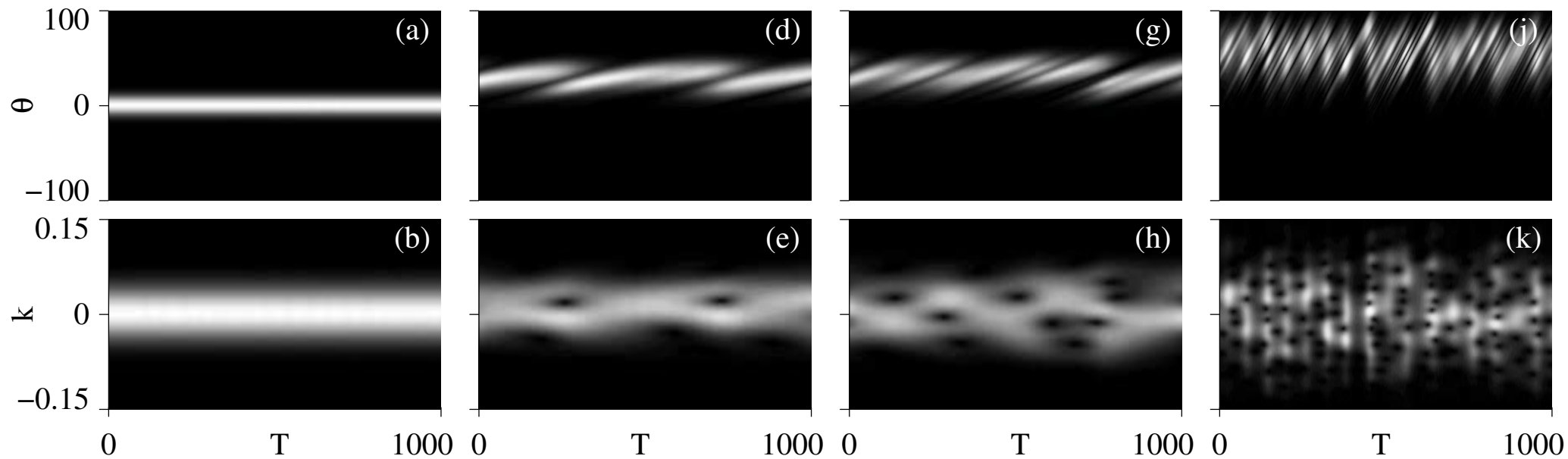
$v=0.5$

$v=0.7$

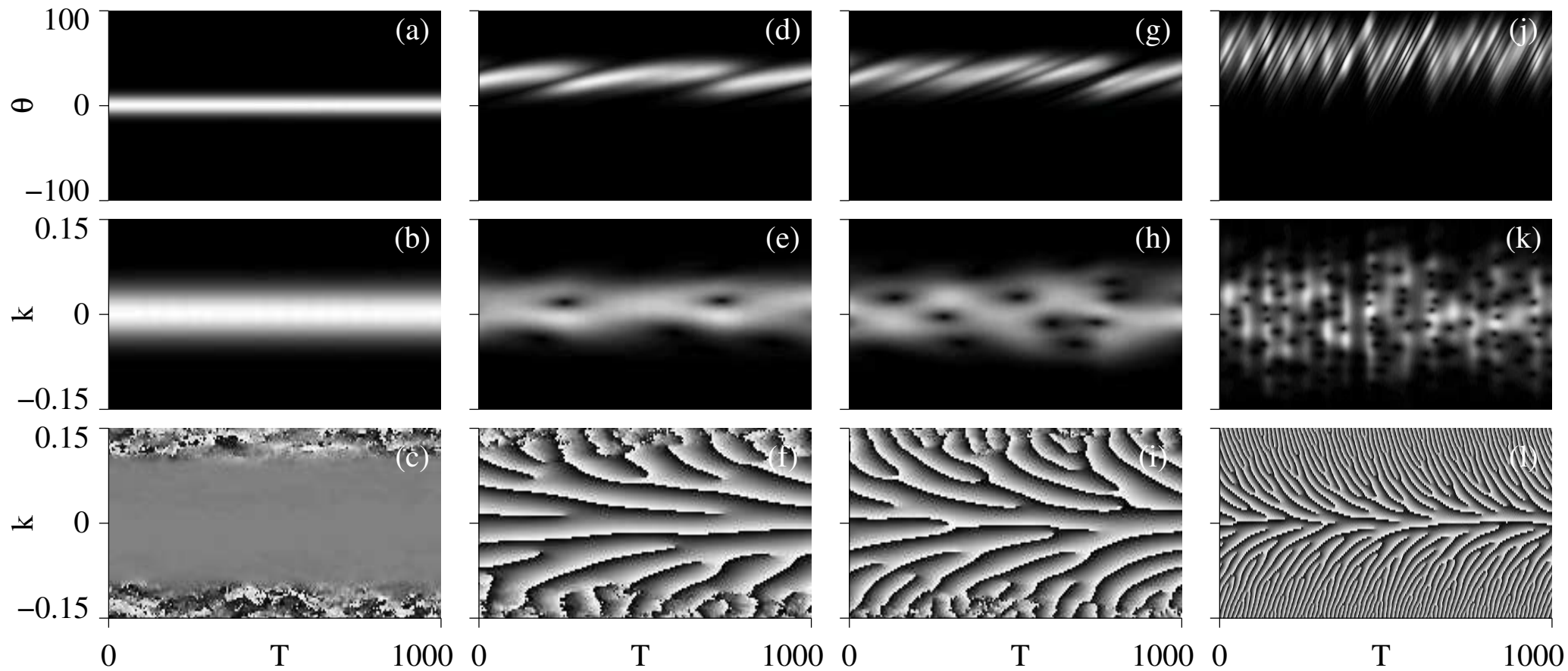
$v=3.4$



# Numerical results

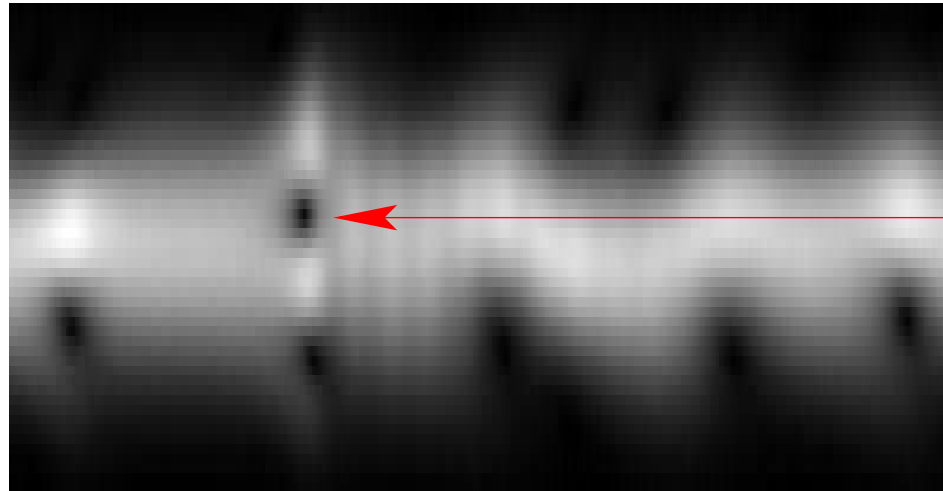


# Numerical results

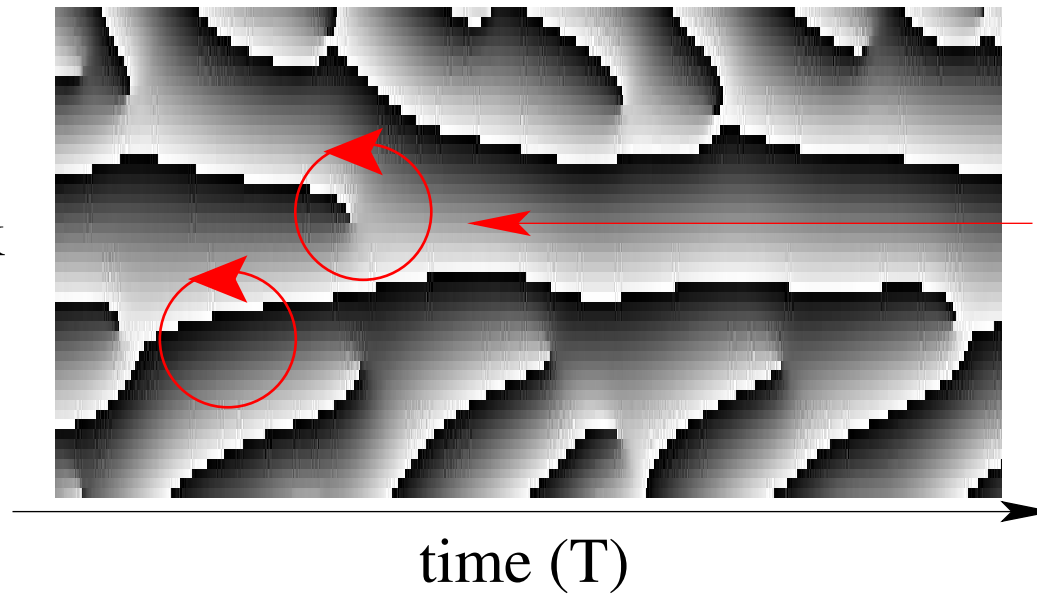


$\tilde{e}(k,T)$   
amplitude

phase

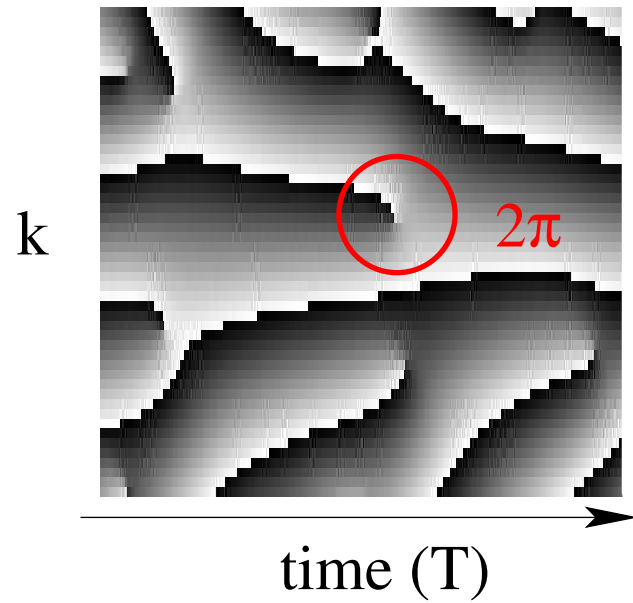
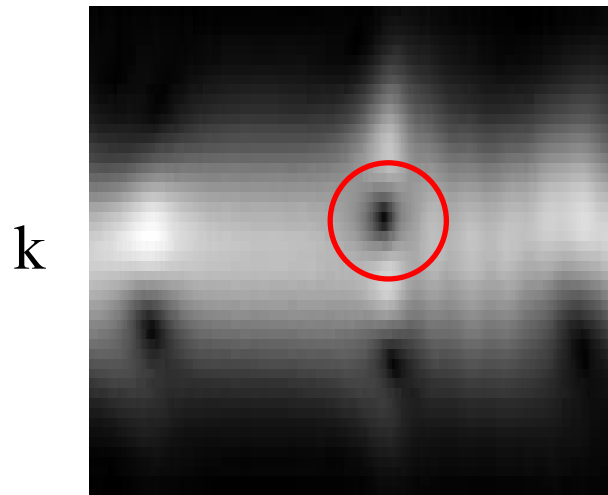


amplitude  
hole:  
 $e(k,T)=0$



$2\pi$   
phase  
singularity

→ "spatio-temporal defect"

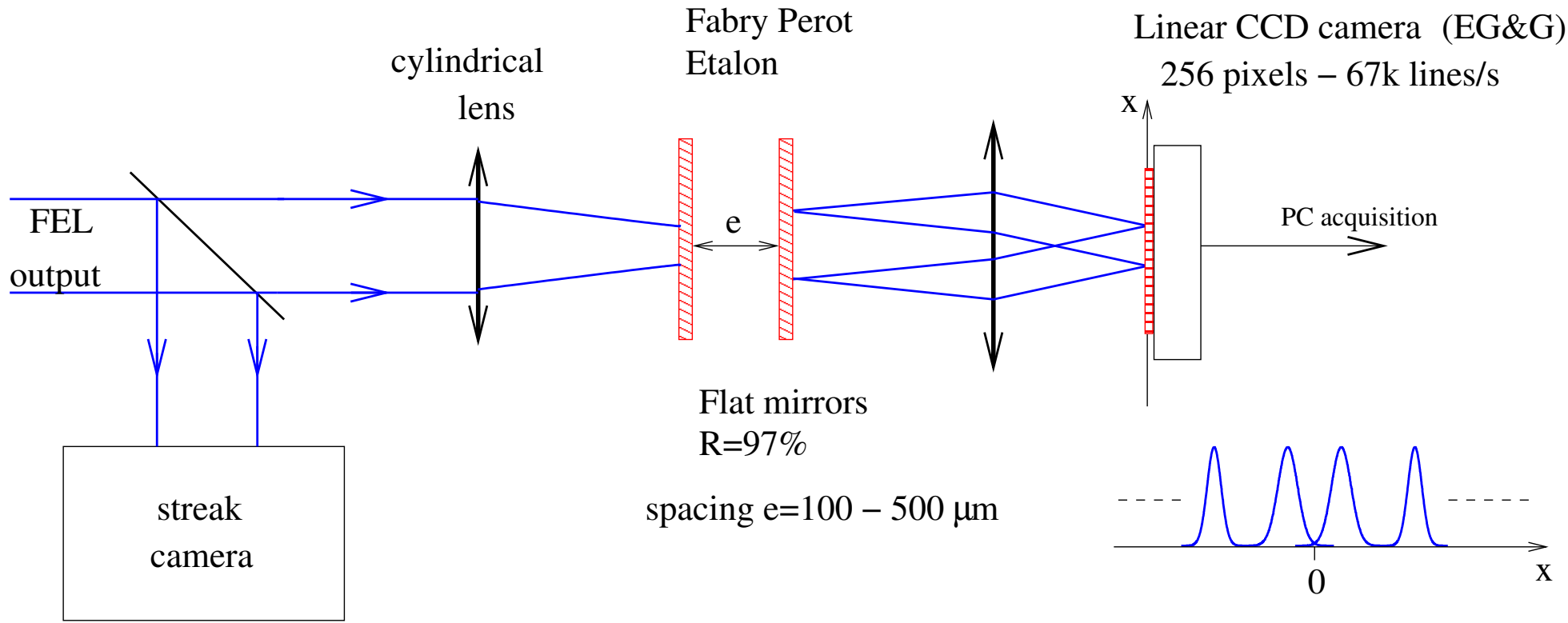


Questions:

Experimentally realistic ?

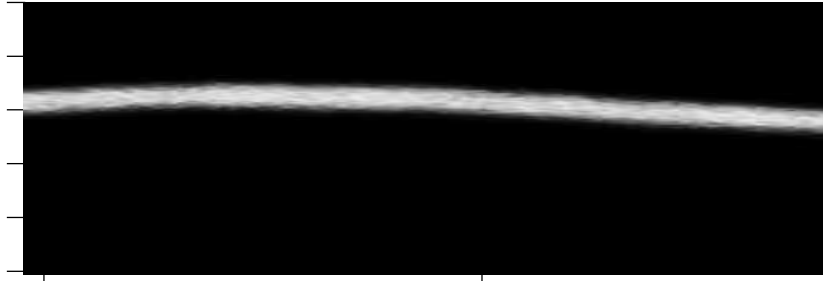
insights on the "origin"  
of these holes ?

Real-time spectrum analyzer



## Experimental results (super-ACO)

Fast time  $\theta$  (25 ps/div.)

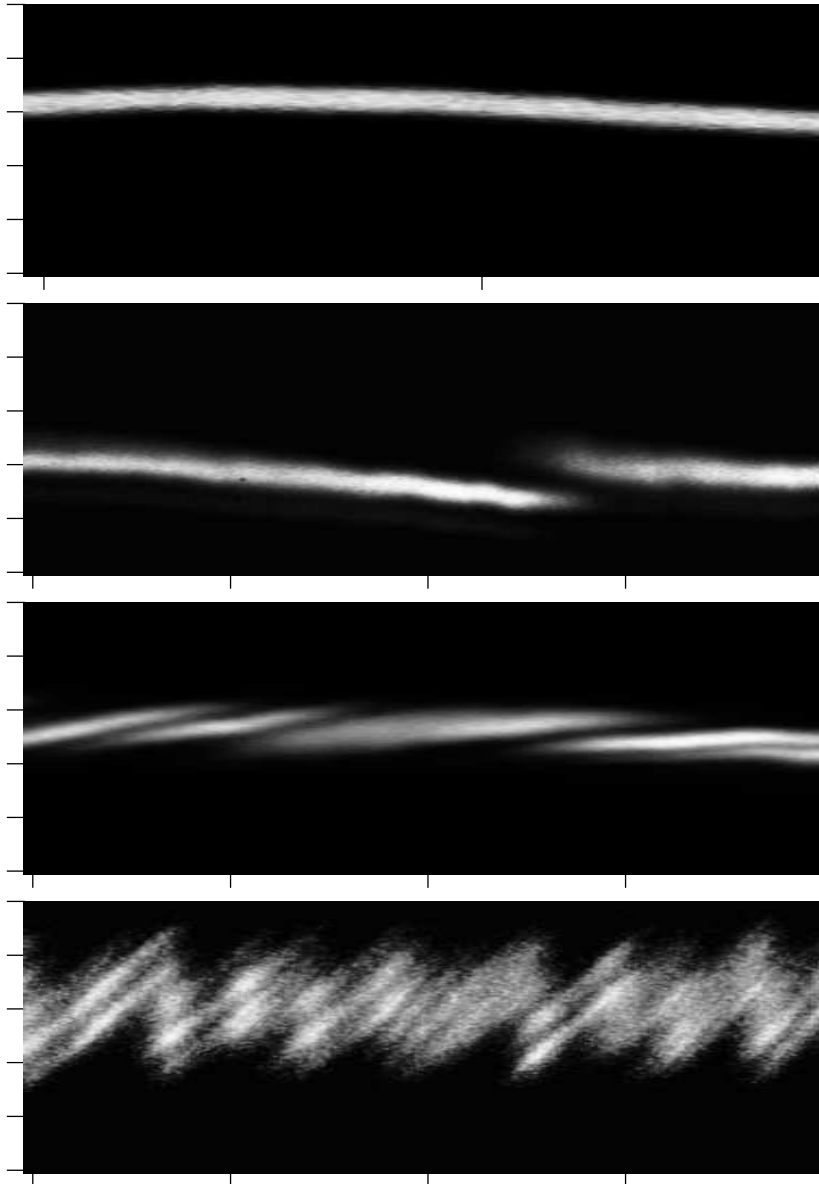


Slow time  $T$  (5 ms/div.)

# Experimental results (super-ACO)

frequency mismatch

Fast time  $\theta$  (25 ps/div.)

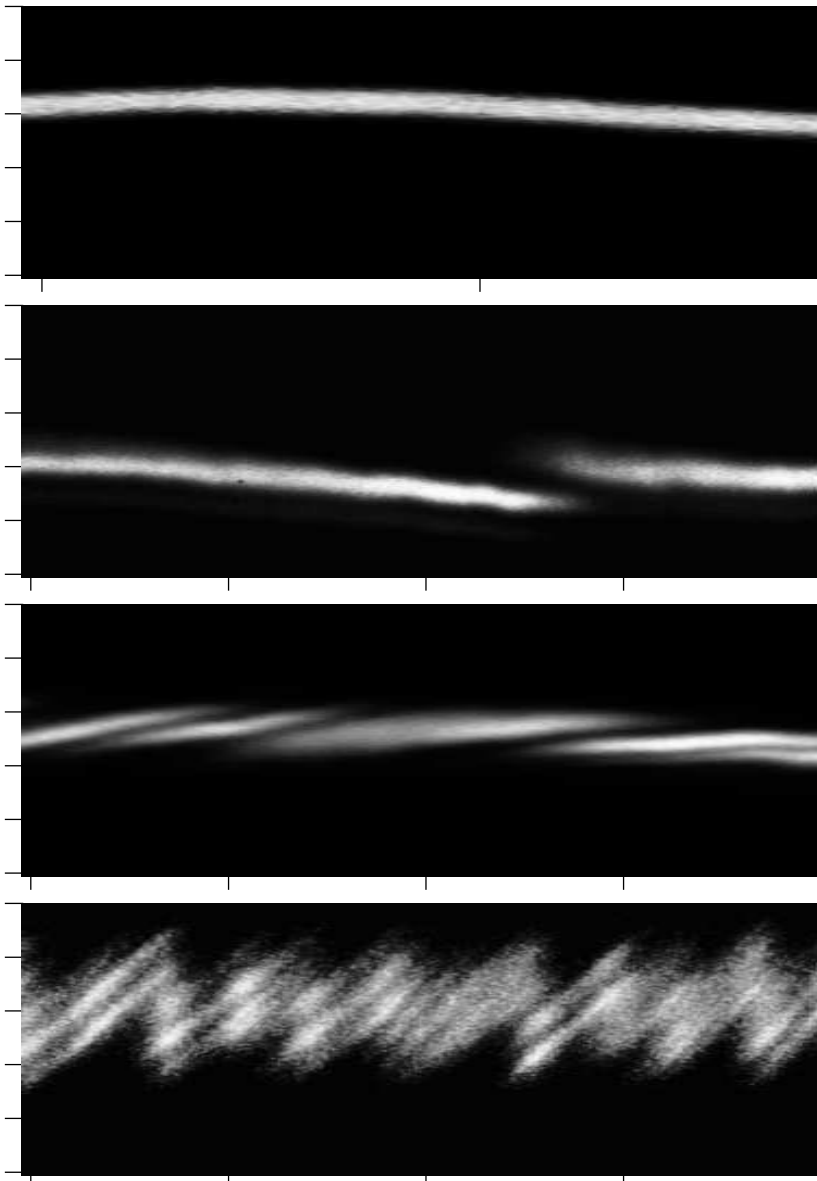


Slow time  $T$  (5 ms/div.)

# Experimental results (super-ACO)

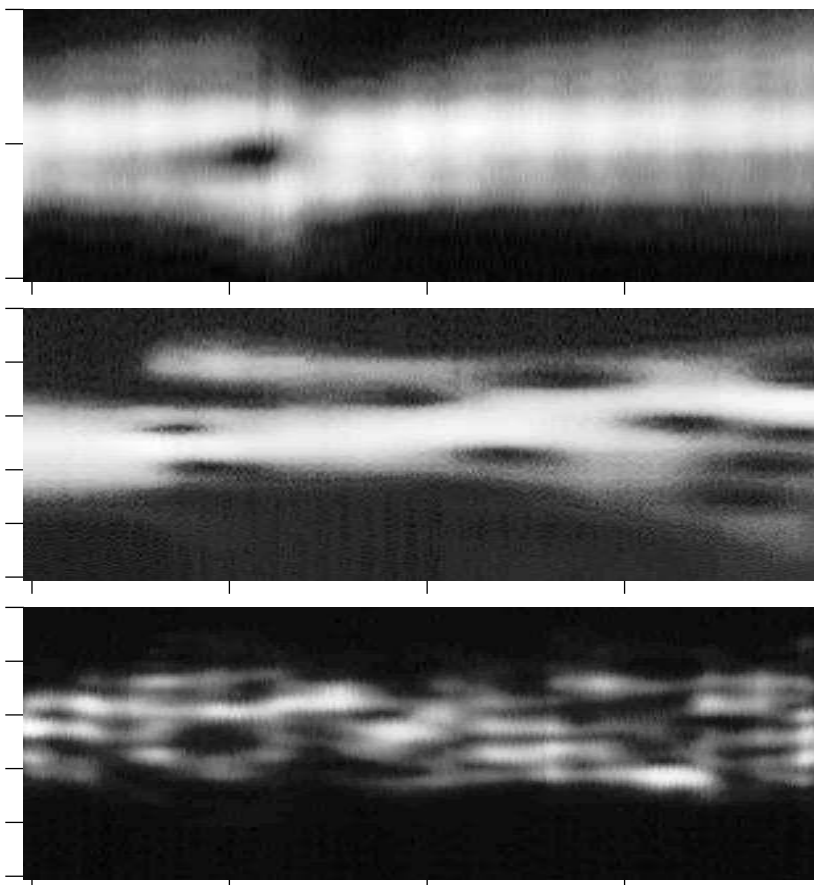
frequency mismatch

Fast time  $\theta$  (25 ps/div.)



Slow time  $T$  (5 ms/div.)

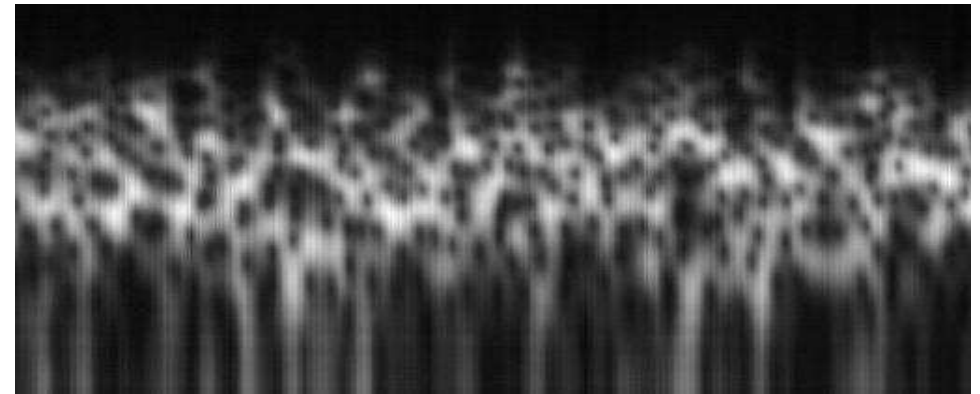
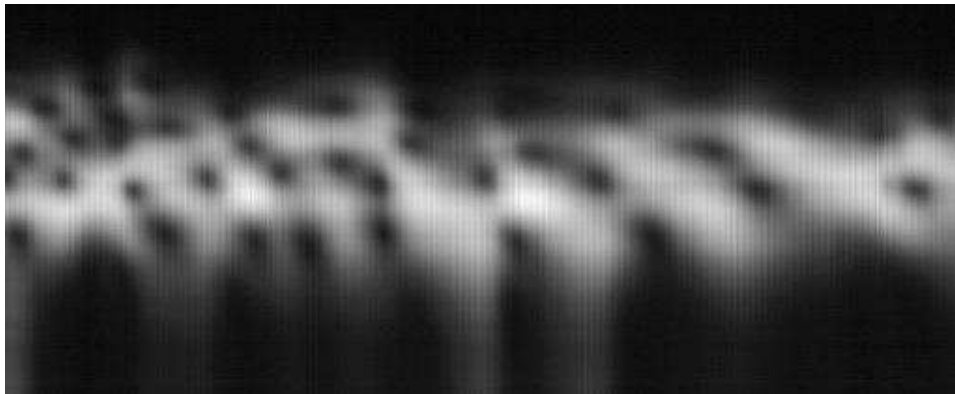
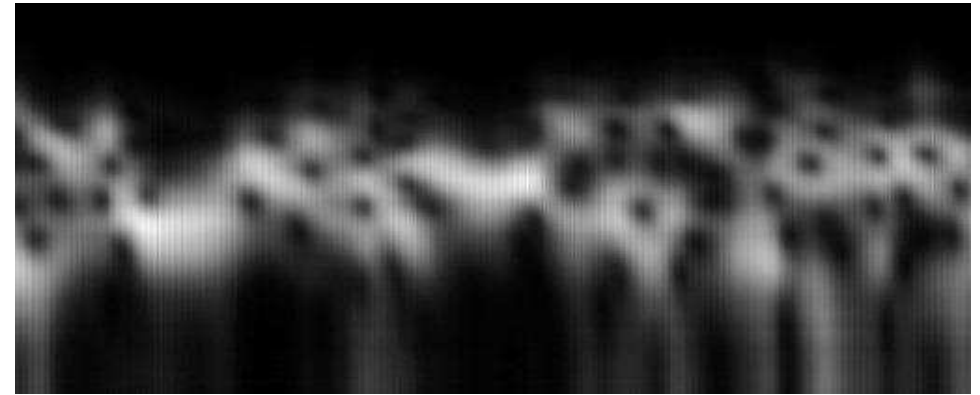
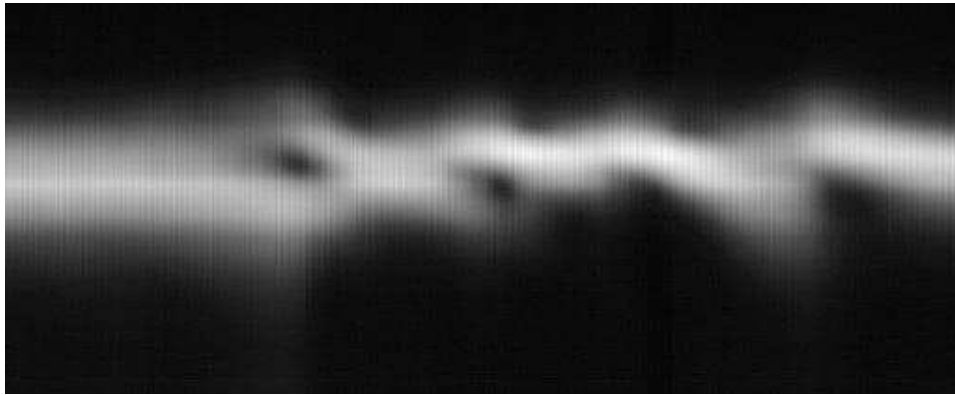
Wavelength (0.25 Å/div.)



Slow time  $T$  (5 ms/div.)



# Optical spectrum versus time: Recent results at UVSOR (IMS, Japan)



time (400 ms/div)

time (400 ms/div)

*Nonlinear dynamics point of view:  
Minimum dynamical ingredients?*

Part of this specific model is necessary for the instability, part is NOT

$$e_T + ve_\theta = -e + g(T)f(\theta)(e + e_{\theta\theta}) + \eta\xi, \quad (1)$$

$$g(T) = \frac{A}{\sigma^2(T)} \exp [-(\sigma^2(T) - 1)/2] \quad (2)$$

$$\text{with } \frac{d\sigma^2}{dT} = \gamma \left( 1 - \sigma^2 + \int_0^L |e(\theta, T)|^2 d\theta \right). \quad (3)$$

- laser pulse length  $\ll$  bunch length: Taylor expansion of  $f(\theta)$
- identification of slowest timescales? Usually  $\gamma$  (e.g., macropulse instabilities). Here?

## "Minimal" equations ?

A Ginzburg-Landau equation with

- global coupling
- a slowly-varying parameter

$$e_T + v e_z = e_{zz} + R [1 - (\epsilon z)^2] e - e \int_{-\infty}^{+\infty} |e|^2 dz + \eta \xi$$

or  $-|e|^2 e$



convection

(Global  
or local  
coupling)

# "Minimal" equations ?

A Ginzburg-Landau equation with

- global coupling
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$$e_T + ve_z = e_{zz} + R [1 - (\epsilon z)^2] e - e \int_{-\infty}^{+\infty} |e|^2 dz + \eta \xi$$

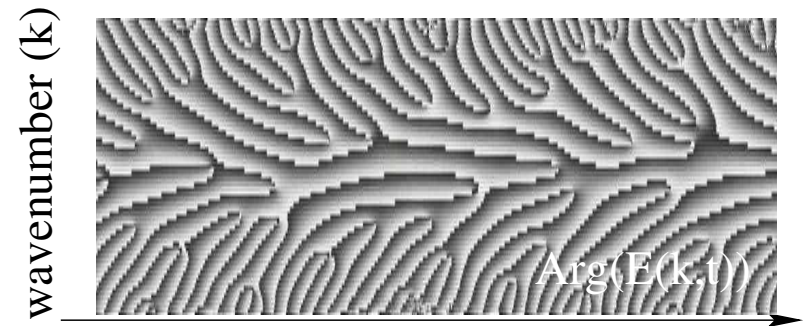
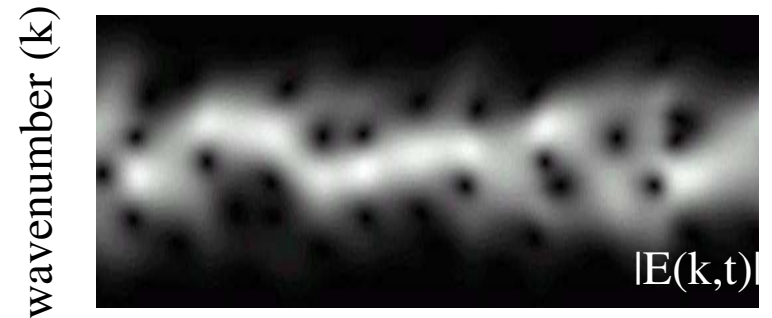
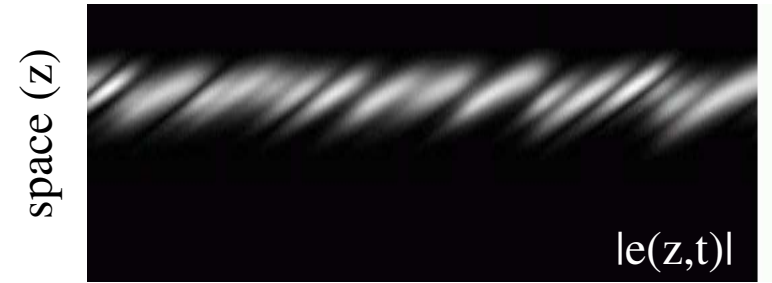


convection

or  $-|e|^2 e$

(Global or local coupling)

ex. with global coupling



T

# Mechanism ? (GL+global coupling)

A Ginzburg-Landau equation with

- global coupling
- a slowly-varying parameter

$$e_T + ve_z = e_{zz} + R [1 - (\epsilon z)^2] e - e \int_{-\infty}^{+\infty} |e|^2 dz + \eta \xi$$

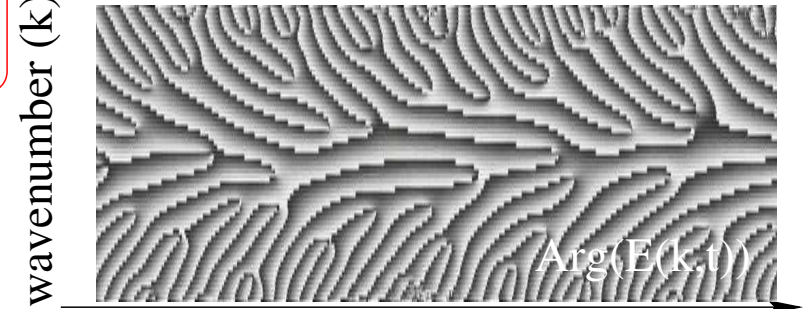
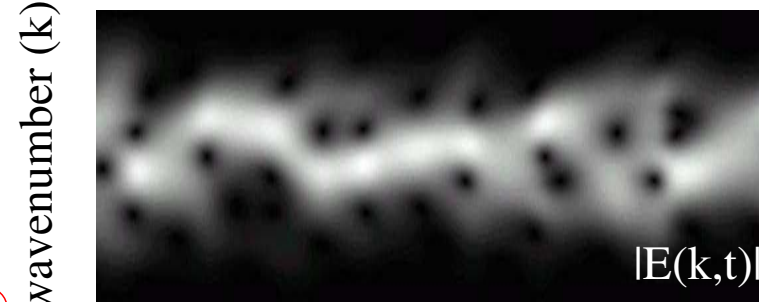
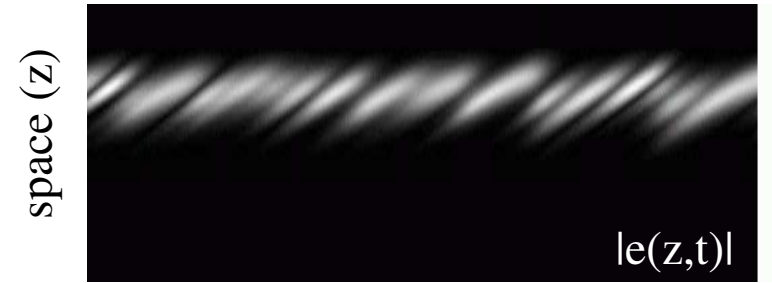
In Fourier space:

$$E_T = \epsilon^2 E_{kk} + RE - (ivk + k^2)E - E \int_{-\infty}^{+\infty} |E|^2 dk + \eta \xi$$

Diffusion

**Non-uniformities  
of control parameters**

Global coupling



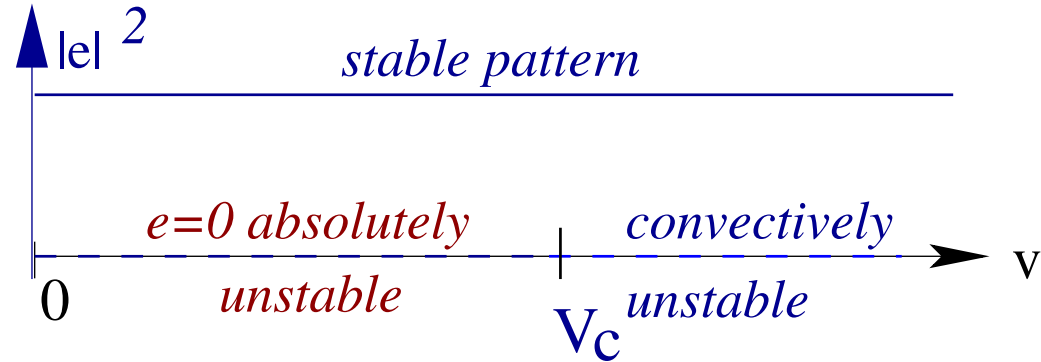
T

Open question: links with the "Riecke and Paap instability? Riecke and Paap, PRL 59, 2570 (1987)

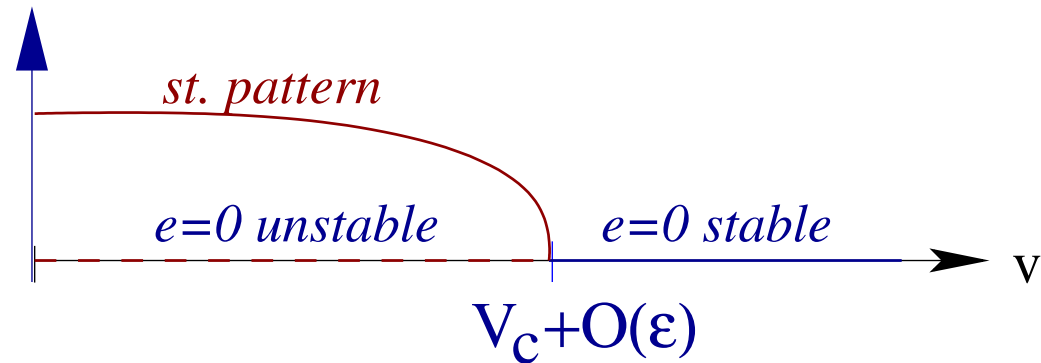
Differences between global and local couplings?  $\rightarrow$  (1) local

$$e_T + v e_z = e_{zz} + R [1 - (\epsilon z)^2] e - |e|^2 e + \eta \xi$$

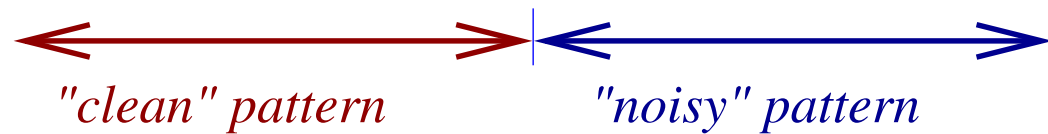
*infinite medium ( $\epsilon=0$ )*



*finite medium ( $\epsilon \neq 0$ )*



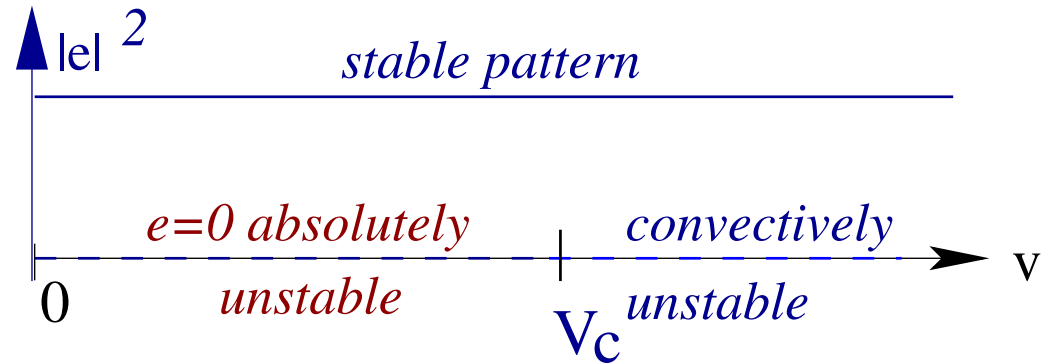
*finite medium  
+noise*



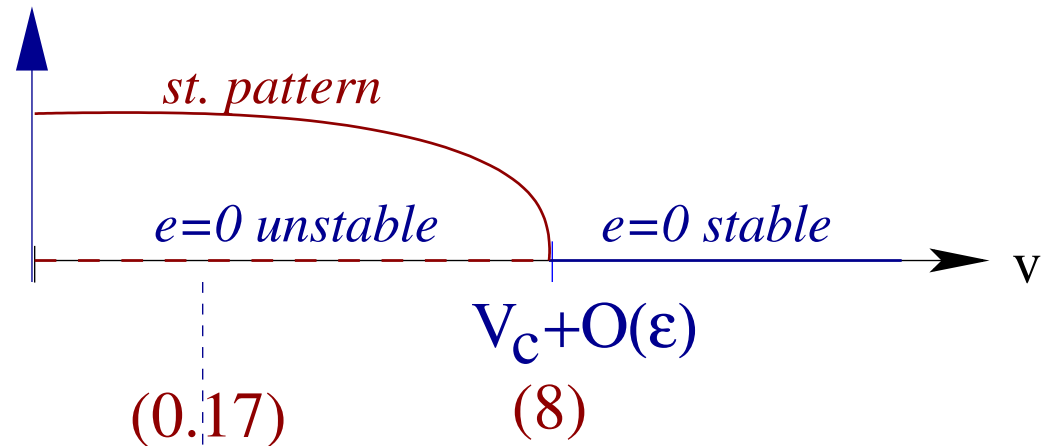
Differences between global and local couplings?  $\rightarrow$  (2) global

$$e_T + v e_z = e_{zz} + R \left[ 1 - (\epsilon z)^2 \right] e - e \int_{-\infty}^{+\infty} |e|^2 dz + \eta \xi$$

*infinite medium ( $\epsilon=0$ )*



*finite medium ( $\epsilon \neq 0$ )*

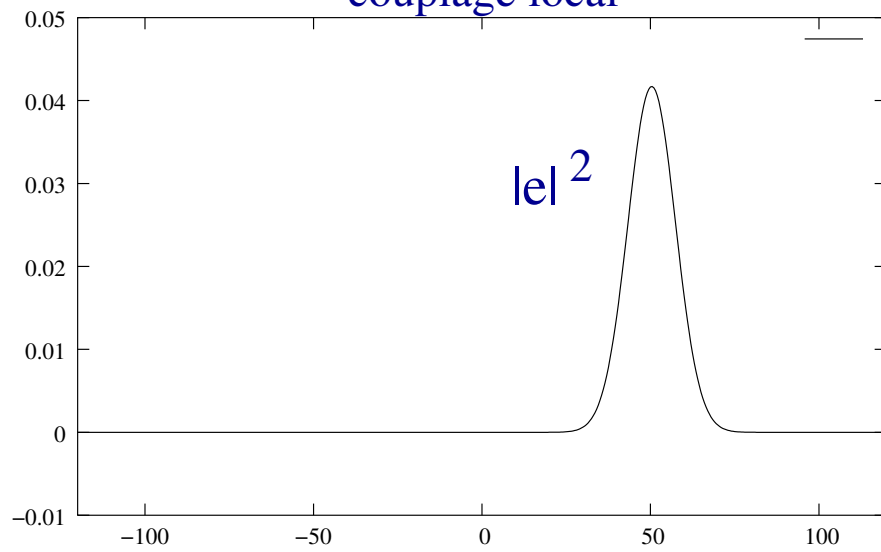


*finite medium  
+noise*

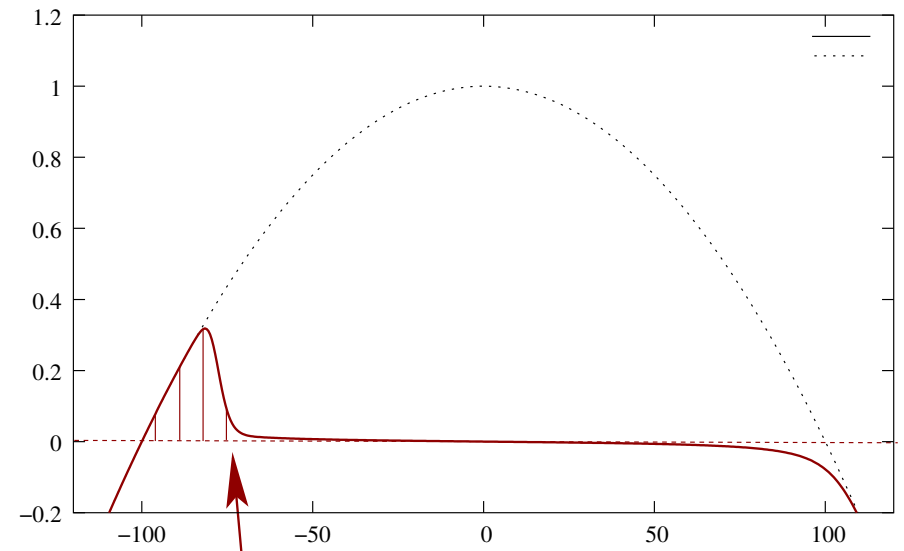
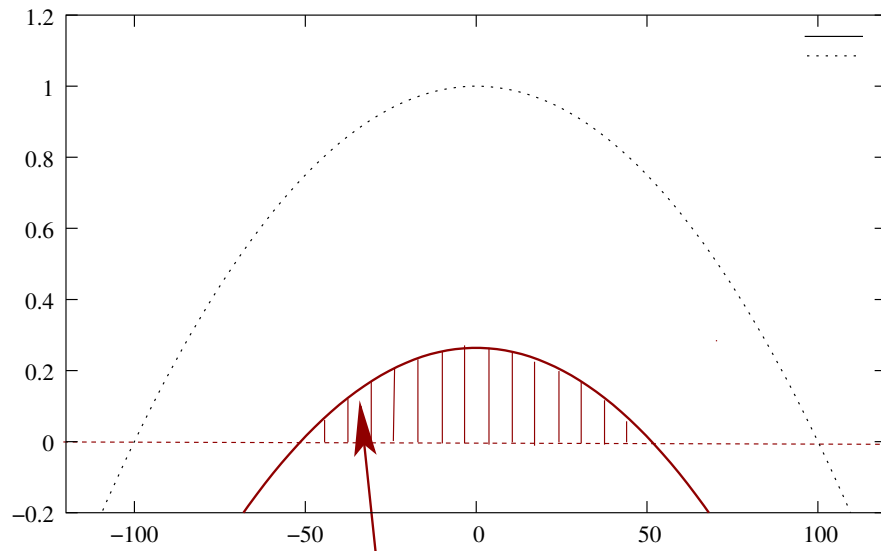
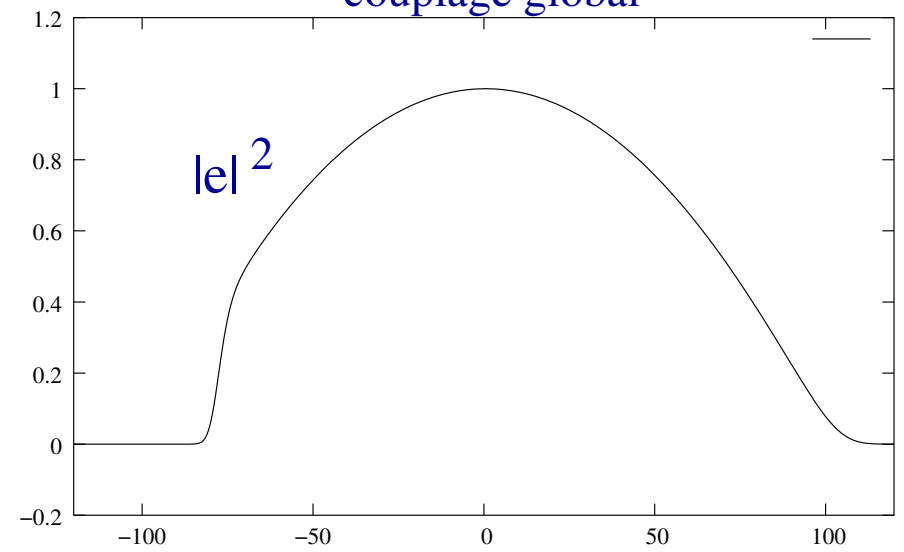
*"clean" pattern*

*"noisy" pattern*

couplage local



couplage global



$$e_T + ve_z = e_{zz} + \left( R [1 - (\epsilon z)^2] - \int_{-\infty}^{+\infty} |e|^2 dz \right) e$$

$$e_T + ve_z = e_{zz} + \left( R [1 - (\epsilon z)^2] - |e|^2 \right) e$$



## *Conclusion*

- \* Laser à électrons libres = système avec advection + saturation globale
- \* Transition lorsque  $v$  augmente  $\rightarrow$  trous spectro-temporels  
 $\rightarrow$  idem pour Ginzburg–Landau avec couplage local ou global

Bielawski, Szwaj, Bruni, Garzella, Orlandi, Couprie, PRL 95, 034801 (2005)

*For other issues (FEL control), see:*

Bielawski, Bruni, Garzella, Orlandi, Couprie, PRE, 69, R045502 (2004)

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**couplage  
global**

Distinction entre:

- Domaines convectif/absolu
- Structures entretenues par le bruit
- Structures bruyantes

$\rightarrow$  *poster RNL*

Bielawski, Szwaj, Bruni, Garzella, Orlandi, Couprie, PRL 95, 034801 (2005)

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$$e_T + ve_z = e_{zz} + R [1 - (\epsilon z)^2] e - e \int_{-\infty}^{+\infty} |e|^2 dz + \eta \xi$$


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$$|e|^2 e$$

