FORMATION OF A VORTEX LATTICE

IN ROTATING BOSE AND FERMI GASES

Y. Castin, LKB, Ecole Normale Supérieure, Paris

with S. Sinha, C. Lobo, A. Sinatra, G. Tonini, F. Werner

Experiments at ENS:

- **rotating Bose gases: J. Dalibard**
- **strongly interacting Fermi gases: C. Salomon, F. Chevy**

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OUTLINE

- **Brief overview of the quantum gases**
- **What is the mechanism of formation of vortex lattices ?**
  - **Experimental verdict**
  - **Hydrodynamic theory**
- **Numerical studies:**
  - **bosons: time dependent non-linear Schrödinger equation**
  - **fermions: time-dependent BCS equations**

BRIEF OVERVIEW OF THE QUANTUM GASES

A little bit of history:

- 1988, the ancestors: the laser cooled gases (Chu, Phillips, Cohen-Tannoudji)
- 1995, the birth: Bose-Einstein condensate of atoms (Cornell, Wieman, Ketterle)
- 2002, the second revolution: superfluid Fermi gases (Thomas, Salomon, Jin, Grimm, Ketterle, Hulet)

Typical experimental parameters:

- $N \sim 10^6$ atoms in magnetic or laser traps
- densities $\rho \sim 10^{13}\ \mathrm{at/cm}^3 \sim 10^{-6}\rho_{\mathrm{air}}$
- temperatures $T \sim 1\mu K$ by evaporation
- gases: $\rho^{-1/3} \gg$ interaction potential range

BRIEF OVERVIEW OF THE QUANTUM GASES (2)

A unique feature: control of the interaction strength

- at low T, interaction characterized by scattering length a
- Fano-Feshbach resonance: a adjusted by magnetic field
- bosons: stable in weakly interacting regime $a \ll \rho^{-1/3}$
- fermions: stable even if strongly interacting $|a| \gg \rho^{-1/3}$:

$$T_c \simeq 0.15 T_F \quad \text{(Svistunov)}$$

beats high-Tc supraconductors and neutron stars

HOW TO ROTATE A QUANTUM GAS ?

A Gedanken experiment:

- trap the gas in a harmonic potential:

$$U(\vec{r}) = \frac{1}{2}m\omega^2\left[(1-\epsilon)x^2 + (1+\epsilon)y^2\right] + \frac{1}{2}m\omega_z^2 z^2$$

  with trap anisotropy $\epsilon \ll 1$.

- rotate the trap at angular frequency Ω.
- No thermal bath: original and pure regime ($\neq$ liquid helium in a container)

Can be done in a real experiment (Dalibard, 2000)

- (Almost) axi-symmetric magnetic trap
- Rotating anisotropy produced by a laser beam

WHY ROTATE A QUANTUM GAS ?

What is the equilibrium state of the gas ?

- classical gas: velocity field is solid body rotation

$$\vec{v}(\vec{r}) = \vec{\Omega} \wedge \vec{r}$$

- quantum gas: $\vec{v}$ is gradient of phase of order parameter

$$\mathrm{curl}\, \vec{v} = \vec{0}$$

  except on lines of phase singularities: quantum vortices

- expected to arrange in triangular lattice (Abrikosov)

How does the vortex lattice form ?

- Expected: thermodynamic instability (negative energy mode)

$$\Omega_{\text{lattice}} \geq \Omega_{\text{Landau}}$$

- Experimental verdict: in a narrow interval close to

$$\Omega_0 = 0.7\omega.$$

EXPLANATION FROM HYDRODYNAMIC THEORY

In simple words:

- stirrer acts if it resonantly excites a condensate mode
- hydrodynamic theory predicts mode frequencies (Stringari)
- resonance condition with lowest quadrupolar mode:

$$2\Omega = \sqrt{2}\omega$$

- at resonance, large amplitude oscillations of the condensate, unstable according to hydrodynamic theory (complex energy mode) (Sinha, Castin)

We suggested an experimental test:

- switch on the rotation slowly rather than abruptly (spin-dryer procedure)
- for low ϵ, hydrodynamic instability for $\Omega > 0.778\omega$
- Experiment: vortices for $\Omega \geq 0.77\omega$ (Dalibard)

MORE DETAILS ON HYDRODYNAMIC THEORY

Superfluid hydrodynamic equations in rotating frame:

$$\partial_t \rho = -\mathrm{div}\left[\rho\left(\vec{v} - \vec{\Omega}(t) \wedge \vec{r}\right)\right]$$

$$-\partial_t S = \frac{1}{2}mv^2 + U(\vec{r}) + \mu_0[\rho(\vec{r},t)] - \mu - m(\vec{\Omega}(t) \wedge \vec{r}) \cdot \vec{v}$$

- ρ is the gas density
- $S/\hbar$ is the phase of the order parameter.
- $\vec{v} = \mathrm{grad}\, S/m$ is the superfluid velocity in lab frame
- $\mu_0[\rho]$ is the chemical potential of the $T = 0$ uniform gas:

bosons and 2D fermions:	$\mu_0 \propto \rho$
3D fermions:	$\mu_0 \propto \rho^{2/3}$ for $a = \infty$

- if initial state=stationary state, hydrodynamic equations and linear stability analysis can be solved/performed analytically

SIMULATIONS FOR SPIN-DRYER PROCEDURE

Bosons in 3D: time dependent Gross-Pitaevskii equation

$$i\hbar\partial_t\psi(\vec{r},t) = \left[h_0(t) + \frac{4\pi\hbar^2 a}{m}|\psi(\vec{r},t)|^2\right]\psi$$

with one-body hamiltonian represented by differential operator

$$h_0 = -\frac{\hbar^2}{2m}\Delta + U(\vec{r}) - \mu - \Omega(t)L_z$$

- Quantum vortices are in the complex field ψ
- Directly observable: atomic density $|\psi|^2$
- Vortices seen as holes in the density profile

SIMULATIONS FOR SPIN-DRYER PROCEDURE (2)

Fermions in 2D: time dependent BCS theory

$$
\begin{aligned}
i\hbar\partial_t u_s(\vec{r},t) &= +h_0(t)u_s(\vec{r},t) + \Delta(\vec{r},t)v_s(\vec{r},t) \\
i\hbar\partial_t v_s(\vec{r},t) &= -h_0^*(t)v_s(\vec{r},t) + \Delta^*(\vec{r},t)u_s(\vec{r},t)
\end{aligned}
$$

with self-consistency condition for gap parameter:

$$\Delta(\vec{r},t) = g_0 \sum_s u_s(\vec{r},t)v_s^*(\vec{r},t)$$

- **Quantum vortices are in the complex field $\Delta(\vec{r},t)$**
- **Atomic density is not directly related to Δ:**

$$\rho(\vec{r},t) = 2\sum_s |v_s(\vec{r},t)|^2,$$

but still dips in the density

CONCLUSION

- Vortex lattice formation can be described by conservative equations when turbulent regime activated by dynamic instability
- This turbulent scenario was tested experimentally in Bose condensates
- Other scenarios observed in other experiments (non harmonic stirrer, rotation of non-condensed gas) (Foot, Cornell, Ketterle)
- Superfluid fermions: vortex lattice seen by Ketterle (2005), non harmonic stirrer