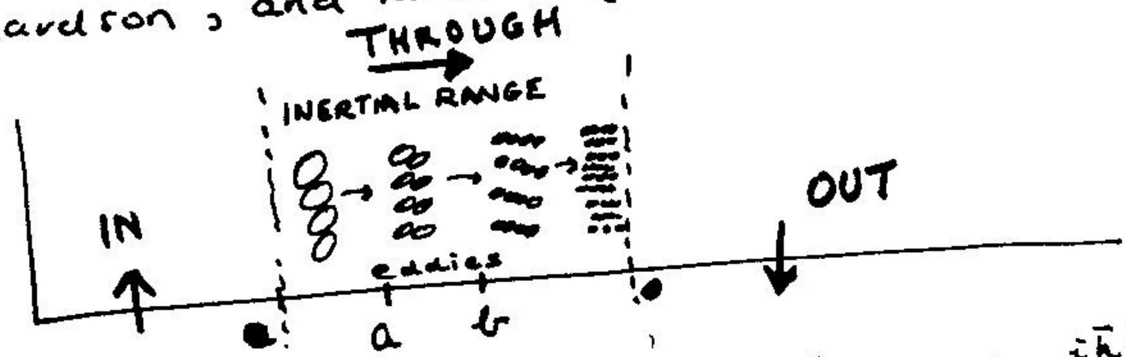
3D hydrodynamic turbulence at large Reynolds' numbers

the granddaddy of turbulent systems

Richardson, and later Taylor, picture



$$R_{ij}(\vec r) = \langle u_i(\vec x)\, u_j(\vec x+\vec r)\rangle = \int \Phi_{ij}(\vec k)\, e^{i\vec k\cdot\vec r}\, d\vec k$$

Avg. energy $E = R_{ii}(0) = \int E(k)\,dk$, $\quad E(k) = \int \Phi_{ii}\, d\Omega$

$$\frac{\partial E}{\partial t} = f(k) + T(k)\left(-\frac{\partial P}{\partial k}\right) - 2\nu k^2 E(k)$$

In inertial range, local energy conservation

$$\frac{\partial}{\partial t}\int_a^b E(k)\,dk = P_a - P_b = \begin{cases} 0 & P_a = P_b = 0 \quad \text{Equipartition} \\ 0 & P_a = P_b = P \quad \text{finite flux Kolmogorov} \end{cases}$$

Two very different kinds of stationary distribution

(i) Isolated system $\Rightarrow$ equipartition

(ii) Non isolated, finite flux $\quad P = \int 2\nu k^2 E(k)\,dk$ finite

major "theorem" for 3D turbulence!

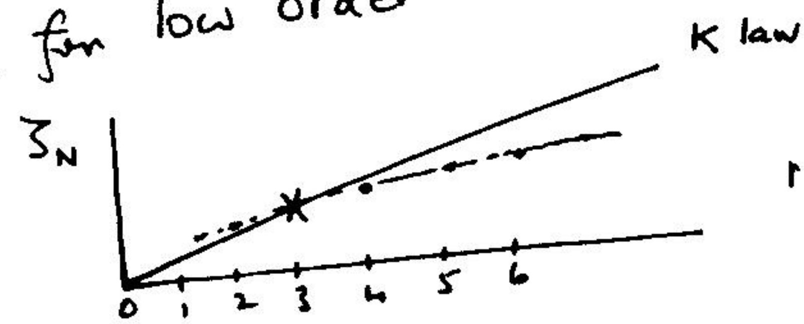
In K '41, Kolmogorov argued that the symmetries of translation / rotation are restored in statistical sense and that, for $Re \to \infty$, all statistical quantities like

$$S_N(r) = \langle (u(\vec{x}+\vec{r}) - u(\vec{x}))^N \rangle = f(r, P)$$

$$\xrightarrow{r\to 0,\, \nu\to 0,\, t\to\infty} (Pr)^{\zeta_N}, \quad \zeta_N = N/3$$

$$\Longleftrightarrow \quad E(k) \sim P^{2/3} k^{-5/3}$$

Good for low order moments but experiments show



$$\lim_{r\to 0} \frac{S_N(r)}{S_2^{N/2}} \sim r^{\zeta_N - \frac{N}{2}\zeta_2} \to \infty$$

$\Rightarrow$ Large fluctuations are more prevalent

Turbulence seems to be a 2 species gas

eddies --- ?

coherent and intermittently .... ?
occurring structures

	3D turbulence	2D turbulence	Wave turbulence
CLOSURE of BBGKY hier.	?	?	Kinetic eqn. + frequency renormalization
SOLUTIONS of BBGKY	? $S_3(r) = \frac{4}{5}Pr$	?	Equipartition + Finite flux + Fronts
MECHANISMS OF TRANSFER	?	?	Wave resonances
STRUCTURES	Eddies? Reconnections?	Vortices? ?	Waves Collapses/whitecaps
WHY INTERMITTENCY?	?	?	Breakdown + flux balances
HOW STATIONARY DISTRIBUTIONS ARE REALIZED	?	?	infinite capacity ✓✓ finite capacity ✓
APPLICATIONS	Many	...	• GRAVITY/OCEAN WAVES • CAPILLARY WAVES • OPTICAL WAVES • SOUND WAVES • ALFVEN WAVES • CARRIER DISTRIBUTIONS IN SEMICONDUCTORS

ASYMPTOTIC CLOSURE $\left(\begin{array}{l} 0 < \varepsilon << 1, \\ \varepsilon \sim \text{wave slope} \end{array}\right)$

1) $$\frac{dA_k^s}{dt} - i\omega_k^s A_k^s = \sum_{r=2} \varepsilon^{r-1} \int L_{kk_1 \cdots k_r}^{ss_1 \cdots s_r} A_{k_1}^{s_1} \cdots A_{k_r}^{s_r} \delta_{12 \cdots r,0} \, d\vec{k}_{1 \cdots r}$$

$$\langle A_k^s A_{k'}^{s'} \rangle = \delta(\vec{k}+\vec{k}') \, q^{s's}(k), \quad q^{-ss}(k) = n_k + \varepsilon q_1^{-ss}(k) + \cdots$$

$$\langle A_k^s A_{k'}^{s'} A_{k''}^{s''} \rangle = \delta(\vec{k}+\vec{k}'+\vec{k}'') \left\{ q_0^{ss's''}(kk'k'') e^{i(s\omega + s'\omega' + s''\omega'')t} + \varepsilon q_1^{ss's''}(kk'k'') \cdots \right.$$

Form BBGKY hierarchy. Solve iteratively

Choose

2) $$\boxed{\frac{dn_k}{dt} = \varepsilon^2 T_2[n_k]^{\downarrow \frac{1}{\varepsilon^2}} + \varepsilon^4 T_4[n_k]^{\downarrow 1/\varepsilon^4} + \cdots}$$

to keep A.E. for $q^{-ss}(k)$ U valid in t.

Slow behaviors of all higher cumulants are captured by

3) $$\omega_k^s \rightarrow \omega_k^s + \varepsilon^2 \Omega_2^s[n_k] + \cdots$$

WHY NATURAL CLOSURE

- LINEAR DISPERSIVE WAVE PROPAGATION LEADS TO DECAY OF ALL N CUMULANTS, $N \geq 3$, SO THAT WAVE FIELD APPROACHES A NEAR JOINT GAUSSIAN STATE FOR $\omega_k t$ LARGE BUT $\varepsilon^2 \omega_k t$ SMALL.

- NONLINEAR REGENERATION OF NONGAUSSIANITY IS DUE TO RESONANCES AND THESE ARE MANIFESTED IN LOW ORDER PRODUCTS

$$Q^{(3)} \sim Q^{(2)} Q^{(2)}$$

$$Q^{(4)} \sim Q^{(2)} Q^{(2)} Q^{(2)}$$

- ONLY ASSUMPTION. AT INITIAL TIME, PHYSICAL SPACE CUMULANTS DECAY AS SEPARATIONS BECOME LARGE.

Three wave resonances

$$
{}^{s}_{2}\left[n_k\right] = \sum_{s_2} \int \left\{ -3i\, L^{s\,s\,s_2\,-s_2}_{k\,k\,k_2\,-k_2} + 4 \sum_{s_1} \int d\vec{k}_1\, L^{s\,s_1\,s_2}_{k\,k_1\,k_2}\, L^{s_1\,s\,-s_2}_{k_1\,k\,-k_2}\, \delta(\vec{k}_1+\vec{k}_2-\vec{k}) \right.
$$

$$
\left. \left(P\frac{1}{s_1\omega_1+s_2\omega_2-s\omega} - i\pi\,\delta(s_1\omega_1+s_2\omega_2-s\omega) \right) \right\} n_{k_2}\, d\vec{k}_2
$$

$$
{}_{2}\left[n_k\right] = 4\pi \sum_{s_1 s_2} \int \left| L^{s\,s_1\,s_2}_{k\,k_1\,k_2} \right|^2 n_k n_{k_1} n_{k_2} \left(\frac{1}{n_k} - \frac{s_1}{s}\frac{1}{n_{k_1}} - \frac{s_2}{s}\frac{1}{n_{k_2}} \right)
$$

$$
\delta(\vec{k}_1+\vec{k}_2-\vec{k})\; \delta(s_1\omega_1+s_2\omega_2-s\omega)\; d\vec{k}_{12}
$$

$$
E_k = \Omega_d\, k^{d-1}\, \omega_k\, n_k
$$

$$
\frac{\partial E_k}{\partial t} = \Omega_d\, k^{d-1}\, \omega_k\, T_2\left[n_k\right] = -\frac{\partial P}{\partial k}
$$

Stationary solutions $\quad n_k = C k^{-x}$

$$
\Rightarrow \quad C^2 \propto P \quad \& \quad d-1+\alpha+2\gamma_2-2x-\alpha+d+1 = 0
$$

$$
\Rightarrow \quad n_k = C_0\, P^{1/2}\, k^{-\gamma_2-d}
$$

Example: Capillary waves

$\gamma_2 = 9/4$, $\alpha = 3/2$, $d = 2$

$$\Rightarrow \quad n_k = C\,\sigma^{-1/4}\,P^{1/2}\,k^{-17/4}, \quad \sigma = S/\rho$$

$$E_k = C\Omega_d\,\sigma^{1/4}\,P^{1/2}\,k^{-7/4}$$

Note $\int_{k_0}^{\infty} E_k\,dk < \infty \Rightarrow$ finite capacity

Breakdown $\dfrac{t_L}{t_{NL}} = \dfrac{1}{\omega_k}\dfrac{1}{n_k}\dfrac{\partial n_k}{\partial t} \sim \sigma^{-3/4}\,P^{1/2}\,k^{-3/4}$

$\rightarrow 0$ for large k

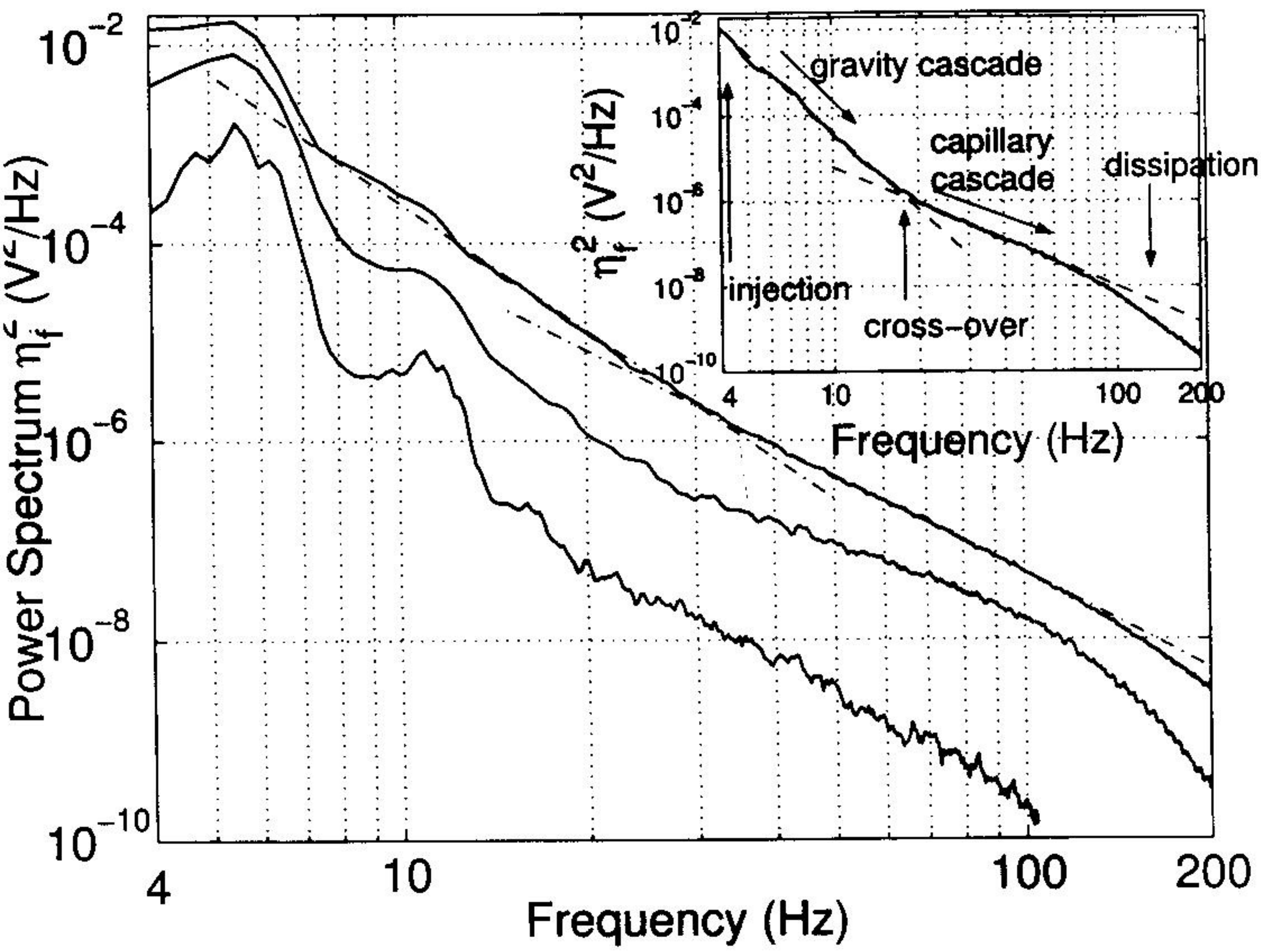
$\rightarrow 1$ for $k_{NL}^{cap} = \dfrac{1}{\sigma}P^{2/3}$

wer spectrum: $S(\Omega) = \dfrac{1}{2\pi}\int \langle \eta(t)\eta(t+\tau)\rangle e^{-i\Omega\tau}\,d\tau \sim \sigma^{1/6}P^{1/2}\Omega^{-17/6}$

Structure functions: $S_2(\tau) = \langle (\eta(t+\tau)-\eta(t))^2 \rangle$

$\sim F\tau^2 + \sigma^{1/6}P^{1/2}\tau^{11/6}$

$S_3(\tau) \sim P\tau^3$, $\quad S_4(\tau) = \underbrace{3S_2^2}_{\text{joint Gaussian}} + \underbrace{\tilde{S}_4}_{\text{non Gaussian part}}$



Power Spectrum η_f^2 (V^2/Hz)
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
4
10
100
200
Frequency (Hz)
η_f^2 (V^2/Hz)
gravity cascade
capillary cascade
dissipation
injection
cross-over

Four wave resonances

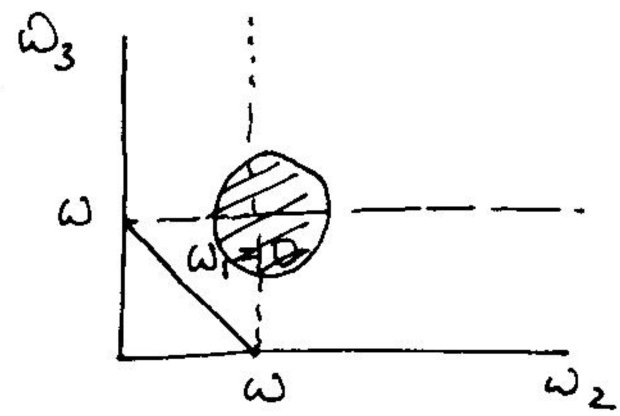
$$T_2[n_k] \equiv 0$$

$$T_4[n_k] = \int \left| L^{+-++}_{k k_1 k_2 k_3} \right|^2 n_k n_{k_1} n_{k_2} n_{k_3} \left(\frac{1}{n_k} + \frac{1}{n_{k_1}} - \frac{1}{n_{k_2}} - \frac{1}{n_{k_3}} \right)$$
$$\delta\left(\vec{k} + \vec{k}_1 - \vec{k}_2 - \vec{k}_3\right) \delta\left(\omega + \omega_1 - \omega_2 - \omega_3\right) d\vec{k}_{123}$$

Angle average $\quad \int N_\omega \, d\omega = \int n_k \, d\vec{k}$

$$\Rightarrow N_\omega = \Omega_d \, k^{d-1} \frac{dk}{d\omega} n_k(\omega)$$

$$\frac{\partial N_\omega}{\partial t} = \int_\Delta S(\omega, \omega_1, \omega_2, \omega_3) n_\omega n_{\omega_1} n_{\omega_2} n_{\omega_3} \left(\frac{1}{n_\omega} + \frac{1}{n_{\omega_1}} - \frac{1}{n_{\omega_2}} - \frac{1}{n_{\omega_3}} \right)$$
$$\delta(\omega + \omega_1 - \omega_2 - \omega_3) \, d\omega_1 \, d\omega_2 \, d\omega_3$$



$$\simeq \frac{\partial^2}{\partial \omega^2} K$$

$$K = S_0 \, \omega^{3x_0 + 2} n_\omega^4 \frac{\partial^2}{\partial \omega^2} \frac{1}{n_\omega}$$

$$x_0 = \frac{2\gamma_3 + 3d}{3\alpha}$$

$$\frac{\partial N_\omega}{\partial t} = \frac{\partial^2 K}{\partial \omega^2}, \quad K = S_0\, \omega^{3x_0+2}\, n^4 \frac{\partial^2}{\partial \omega^2}\left(\frac{1}{n}\right)$$

$$N_\omega = \Omega_d\, k^{d-1} \frac{dk}{d\omega}\, n_k(\omega), \quad x_0 = \frac{2\gamma + 3d}{3\alpha}$$

PARTICLE CONSERVATION

$$\frac{\partial N_\omega}{\partial t} = \frac{\partial \varphi}{\partial \omega}, \quad \varphi = \frac{\partial K}{\partial \omega}$$

ENERGY CONSERVATION

$$\frac{\partial \omega N_\omega}{\partial t} = -\frac{\partial P}{\partial \omega}, \quad P = K - \omega \frac{\partial K}{\partial \omega}$$

ENTROPY PRODUCTION

$$\frac{\partial S}{\partial t} = \frac{1}{n}\frac{\partial N_\omega}{\partial t} = -\frac{\partial R}{\partial \omega} + T$$

R — entropy flux $= K \frac{\partial}{\partial \omega}\frac{1}{n} - \frac{1}{n}\frac{\partial K}{\partial \omega}$

T — bulk production $= S_0\, \omega^{3x_0+2} n^4 \left(\frac{\partial^2}{\partial \omega^2}\frac{1}{n}\right)^2 \geq 0$

STATIONARY SOLUTIONS

(KOLMOGOROV - ZAKHAROV)

$$\frac{\partial^2 K}{\partial \omega^2} = 0 \qquad x_0 = \frac{2\gamma + 3d}{3\alpha} = \begin{matrix} 8 \text{ for gravity wa[ves]} \\ 1 \text{ for optical wa[ves]} \end{matrix}$$

$$\Rightarrow K = S_0 \omega^{3x_0+2} n^4 \frac{\partial^2}{\partial \omega^2} \frac{1}{n} = Q\omega + P$$

where Q, P which can be identified with particle and energy fluxes are constan[t]

In general: $n_R = n_R(\omega; T, \mu, P, Q)$

Special:

$P = Q = 0 \Rightarrow \frac{1}{n} = \frac{1}{T}(\omega + \mu)$ RAYLEIGH -JEANS

$Q = 0$, $T = \mu = 0$: $\quad n = \left(\frac{P}{S_0 x_0 (x_0 - 1)} \right)^{1/3} \omega^{-x_0}$ FINITE ENERGY FLUX

$P = 0$, $T = \mu = 0$: $\quad n = \left(\frac{Q}{S_0 (x_0 - \frac{1}{3})(x_0 - \frac{4}{3})} \right)^{\frac{1}{3}} \omega^{-x_0 + \frac{1}{3}}$ FINITE PARTICLE FLUX

The last two are called KOLMOGOROV - ZAKHAROV SOLNS.

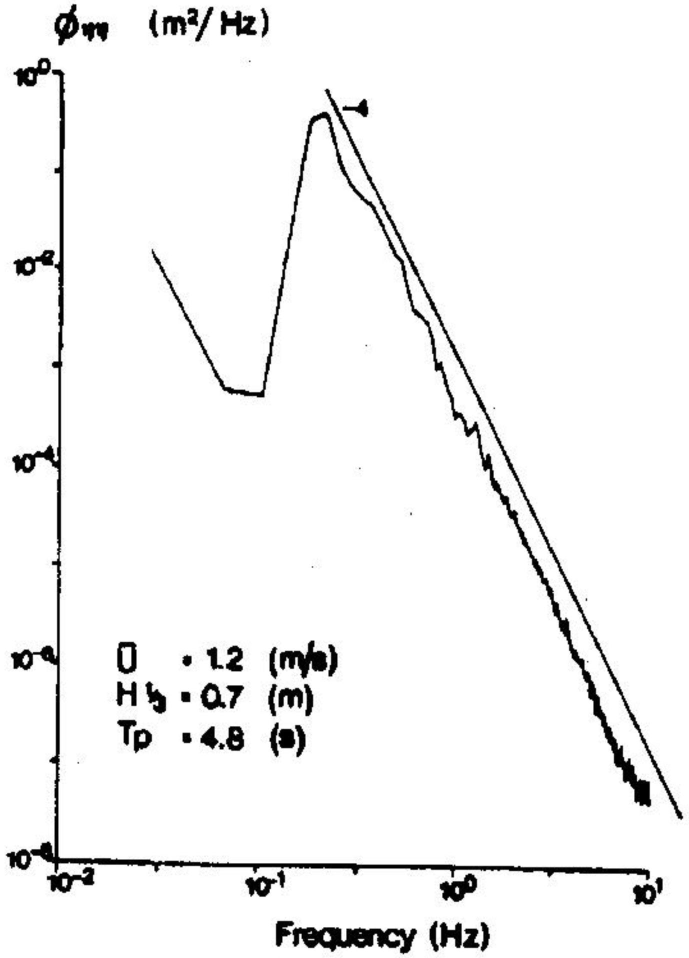
$$\varphi(\omega) \simeq 2.9 \cdot 10^{-3} \frac{g u}{\omega^4}$$

$$E_\omega = \omega k \frac{dk}{d\omega} n_k(\omega)$$

$$= \omega^{1+2+1-8} P^{1/3}$$

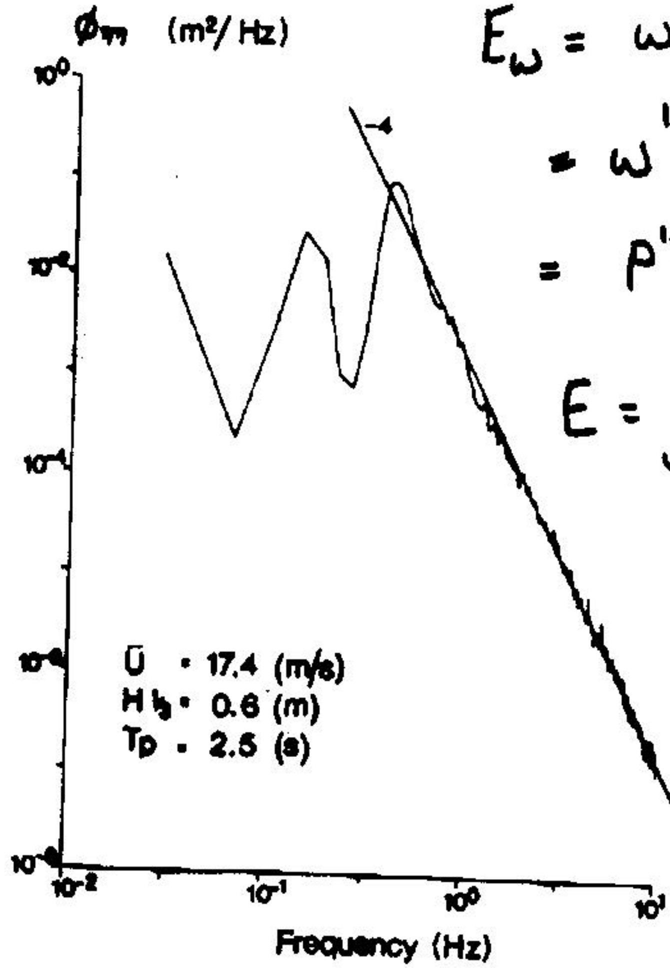
$$= P^{1/3} \omega^{-4}$$

$$E = \int E_\omega \, d\omega$$



RUN 85111

Figure 4a. The spectrum of surface elevation for the case of recent swell. The −4 power law fitted to Figure 4b is also shown here.

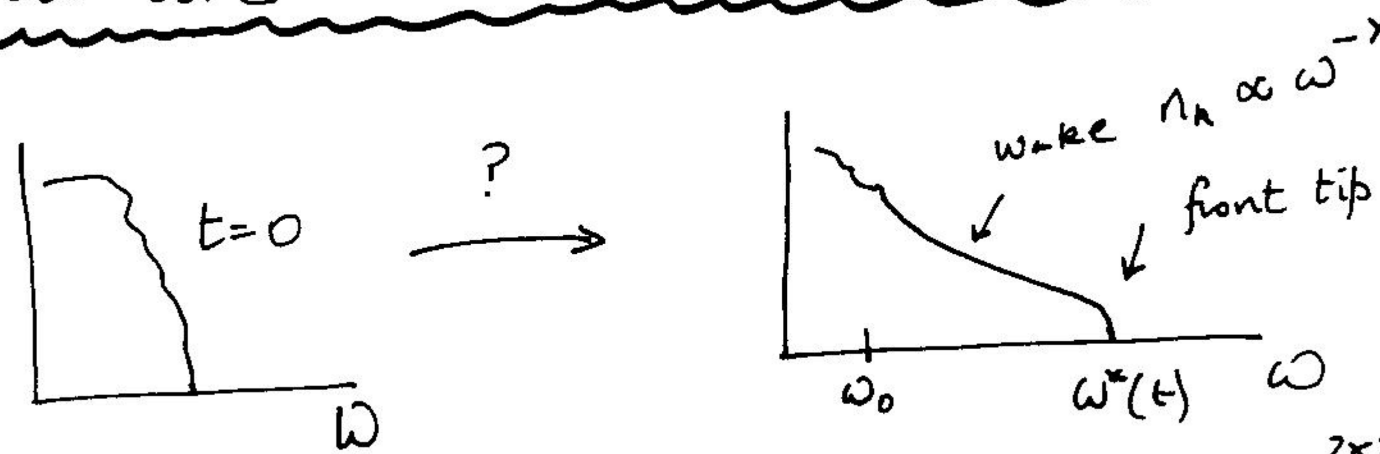


RUN 85169

Figure 4b. The spectrum of surface elevation for the strongly wind-generated case. Above the peak, the spectrum conforms to a −4 power law (Donelan et al, 1985).

$$\varphi(f) = \frac{2 \cdot 10^{-3}}{f^4} = \frac{(2\pi)^4 \cdot 10^{-3} \cdot 2 \cdot U g}{}$$

How are the KZ spectra realized?



Capacity :

$$\int_{\omega_0}^{\infty} \omega N_\omega \, d\omega = \int_{\omega_0}^{\infty} \omega^{-\frac{2\gamma}{3\alpha}} d\omega \begin{cases} < \infty & 2\gamma > 3 \\ = \infty & 2\gamma < 3 \end{cases}$$

$$\int_{0}^{\omega_0} N_\omega \, d\omega = \int_{0}^{\omega_0} \omega^{-\frac{2\gamma}{3\alpha} + \frac{1}{3}} \frac{d\omega}{\omega} \begin{cases} = \infty & 2\gamma > 3 \\ < \infty & 2\gamma < 3 \end{cases}$$

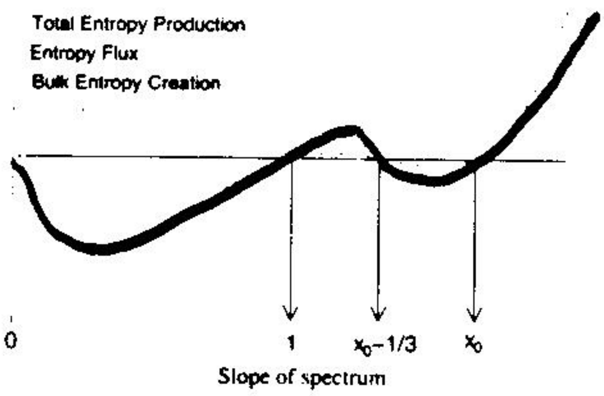
Self similar solution

$$n_k(\omega, t) = \omega^*(\tau)^{-x} F\left(\eta = \frac{\omega}{\omega^*(\tau)}\right)$$

$$\tau = t^* - t \quad \text{or} \quad t$$

$$\omega^*(\tau) \sim \tau^{b}$$

$$b = \frac{1}{2(x - x_0) - \frac{2\gamma - 3\alpha}{3\alpha}}$$



Entropy production as function of x

ig. 1. Plot of the bulk entropy production, T, the entropy flux, R, and the total entropy production, $\dot{S}$, as a function of the exponent, x, of the ... for power law spectra, $n_\omega = c\omega^{-x}$.

$$P = 3Ic^3 x(x-1)(x-(x_0-\tfrac{1}{3}))\omega^{3x_0-3x}, \tag{31}$$

$$T = c^2 Ix^2(x-1)^2\omega^{3x_0-2x-2} \tag{32}$$

$$R = 3c^2 Ix(x-1)(x_0-\tfrac{4}{3}x)\omega^{3x_0-2x-1}, \tag{33}$$

$$\dot{S} = -\frac{\partial R}{\partial\omega} + T = 9c^2 Ix(x-1)(x-x_0)\left(x-\left(x_0-\frac{1}{3}\right)\right)\omega^{3x_0-2x-2}. \tag{34}$$

imilar expressions, which have the *same* zeros (as functions of x) obtain for the case when the differential approx-mation is replaced by the full collision integral. In particular, we note that the entropy production rate is always ositive (we assume $x_0 > 4/3$) for $1 < x < x_0 - 1/3$ and for $x > x_0$, the K–Z exponents for particles and energy, spectively. The relevant functions are plotted in Fig. 1. For $x_0 - 1/3 < x < x_0$, the entropy production rate is egative. This corresponds to a situation when the particle flux is building a condensate state [9].

Numerical observations of non-stationary spectra

Let us now turn our attention to non-stationary solutions of (13). In particular, we are interested in how the K–Z ectra are set up if we begin from an initial condition which is compactly supported at low frequencies. Early work this question focussing on the direct cascade by Falkovich and Shafarenko [5] suggested that the K–Z spectrum set up by a nonlinear front which propagates to the right leaving the K–Z spectrum in its wake. Recent numerical idies by Galtier et al. [6] suggest that this problem is more subtle. They found that in the case where the K–Z ergy spectrum has finite capacity, the approach to the steady-state proceeds by a different mechanism. For this stem the nonlinear front reaches infinite frequency within a finite time, t^*. They found that the quasi-stationary ectrum in the wake of this front was actually steeper than the K–Z spectrum. The K–Z spectrum was then set up m right to left after the front reached infinity.

We investigated whether there was evidence of this behaviour in the solutions of the differential kinetic equation. solved the differential kinetic equation numerically and followed the evolution from an initial condition com-tly supported at low frequencies. The results presented in this section were obtained by allowing the initial data decay freely in the absence of forcing or damping. Some details of the numerical methods used are contained in pendix B.

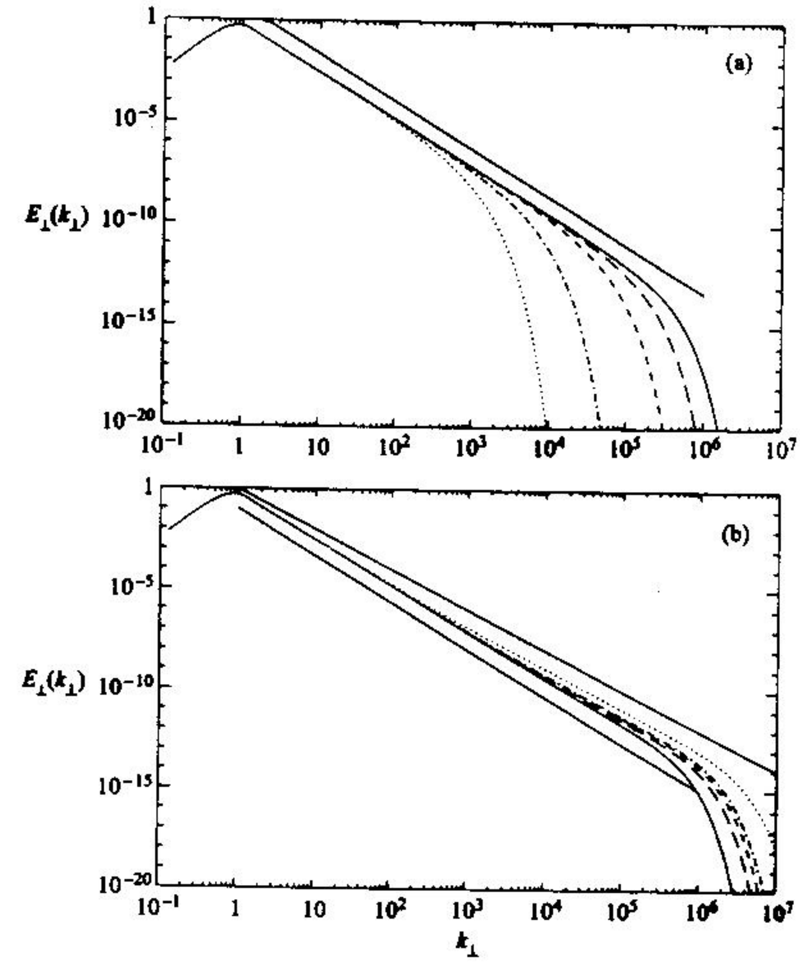


Figure 12. Temporal evolution of the energy spectrum $E_\perp(k_\perp, 0)$ of the shear-Alfvén waves around the catastrophic time $t_0 \approx 1.544$. (a) For $t < t_0$, with $t = 1.50$ (dots), 1.53 (dash–dots), 1.54 (short dashes), 1.542 (long dashes) and 1.543 (solid), a $k_\perp^{-7/3}$ spectrum is observed. (b) For $t \geqslant t_0$, with $t = 1.544$ (solid), 1.546 (long dashes), 1.548 (short dashes), 1.55 (dash–dots) and 1.58 (dots), a fast change of the slope appears to give finally a $k_\perp^{-2}$ spectrum. Note that this change propagates from small scales to large scales. In both cases, straight lines follow either a $k_\perp^{-7/3}$ or a $k_\perp^{-2}$ law.

It is only when the dissipative scale is reached, at t_0, that a remarkable effect is observed: in a very fast time, the $k_\perp^{-7/3}$ solution turns into the finite energy flux spectrum $k_\perp^{-2}$, with a change of the slope propagating from small scales to large scales.

Note that this picture is different from the scenario proposed by Falkovich and Shafarenko (1991, hereinafter FS) for the finite-capacity spectra. In an example considered by FS, the Kolmogorov spectrum forms right behind the propagating front, whereas in our case it forms only after the front reaches infinite wavenumbers (i.e. the dissipative region). The front propagation can be described in terms of self-similar solutions having the form (Falkovich and Shafarenko 1991; Zakharov et al. 1992):

$$E_\perp(k_\perp, 0) = \frac{1}{\tau^a} E_0\left(\frac{k_\perp}{\tau^b}\right), \tag{75}$$

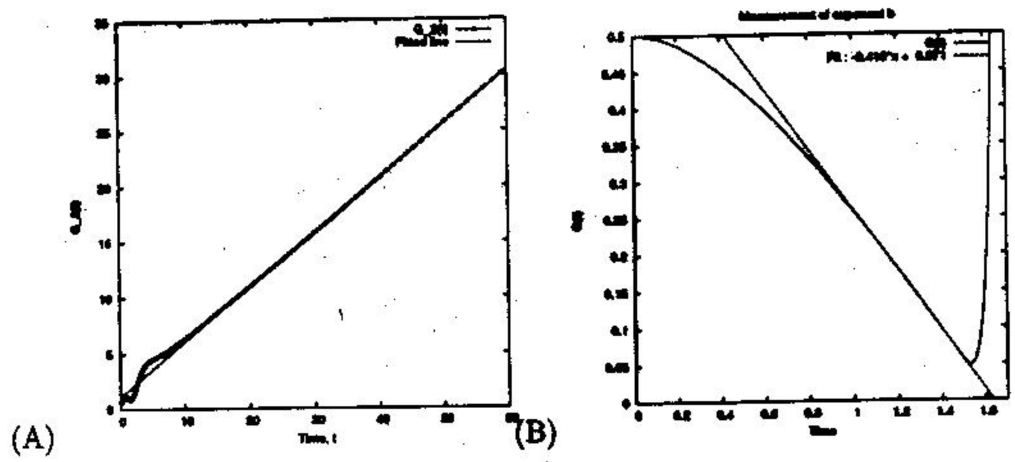


Fig. 3. Measuring the dynamical exponent b for (A) infinite capacity case, $\lambda = 0$ and (B) finite capacity case, $\lambda = 2$..

Figure 2 shows the numerical solutions for an infinite capacity case and a finite capacity case. Both show the propogation of the front described here with power law scaling in the wake. We now turn to the numerical measurement of the dynamical exponents a and b.

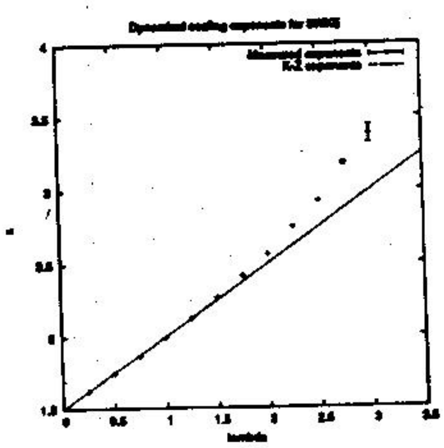


Fig. 4. Exponent, x, of the transient spectrum for a range of values of λ.

6 Conclusions

We conclude that the finite capacity anomaly is a genuine feature of the transient spectrum for finite capacity systems. It is not a by-product of some peculiarity of the MHD equations studied by Galtier et al or of the rather drastic differential approximation which we used to study this phenomenon in earlier work [4].

Infinite capacity case: $E \sim t$

$$X = X_0 = \frac{2\gamma}{3\alpha} + \frac{d}{\alpha}, \quad b = \frac{3\alpha}{3\alpha - 2\gamma} > 0$$

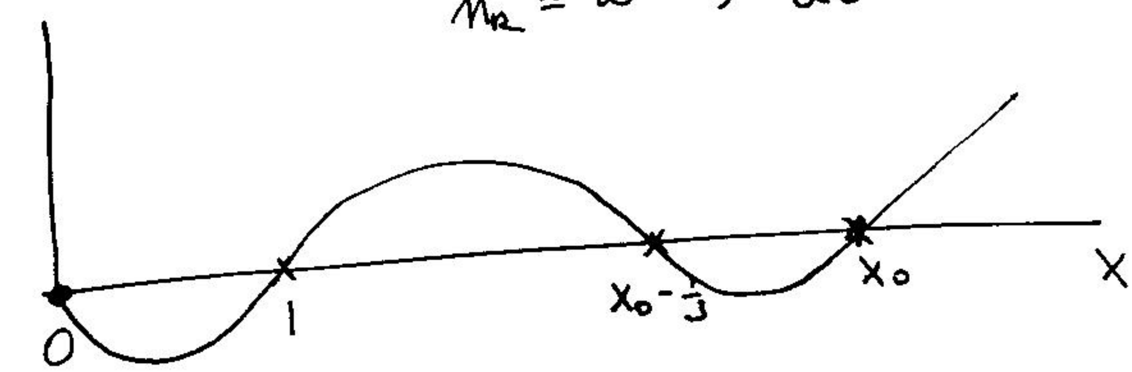
Finite capacity case: $E = O(1)$

$$\begin{cases} \text{Entropy production +ive} \Rightarrow x > x_0 \\ \text{Finite time access to } \omega = \infty \Rightarrow b < 0 \end{cases}$$

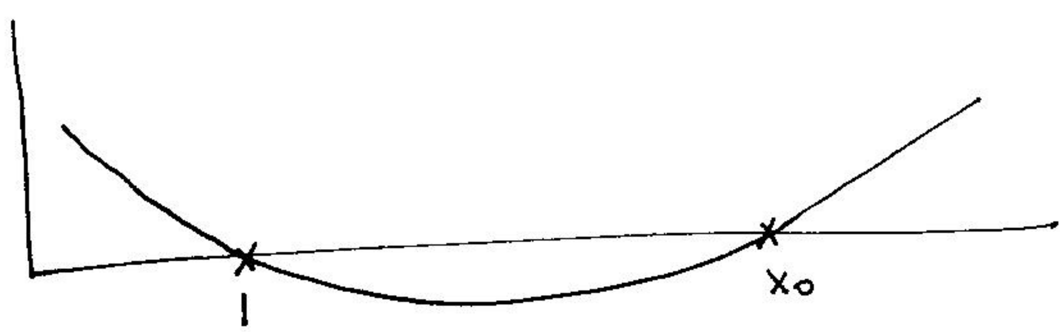
$$\Rightarrow \quad x_0 < x < x_0 + \frac{2\gamma - 3\alpha}{6\alpha}$$

$$n_k = \omega^{-x}, \quad \frac{dS}{dt} = \omega$$

4 wave:



3 wave



The spectra of gravity/capillary waves

Define $k_0 = \left(\frac{g}{\sigma}\right)^{1/2}$, $P_0 = (\sigma g)^{3/4}$, $\sigma = S/\rho$

$$E_{\vec{k}} = \omega_k n_k$$

$$g^{-1} k_0^4 \, E_{\vec{k}}^{(g)} = C \left(\frac{P}{P_0}\right)^{1/3} \left(\frac{k}{k_0}\right)^{-7/2}$$

Phillips

η_x discont

$\Rightarrow k \eta_k \sim 1/k$

$\Rightarrow \eta_k^2 \sim \frac{1}{k^4}$

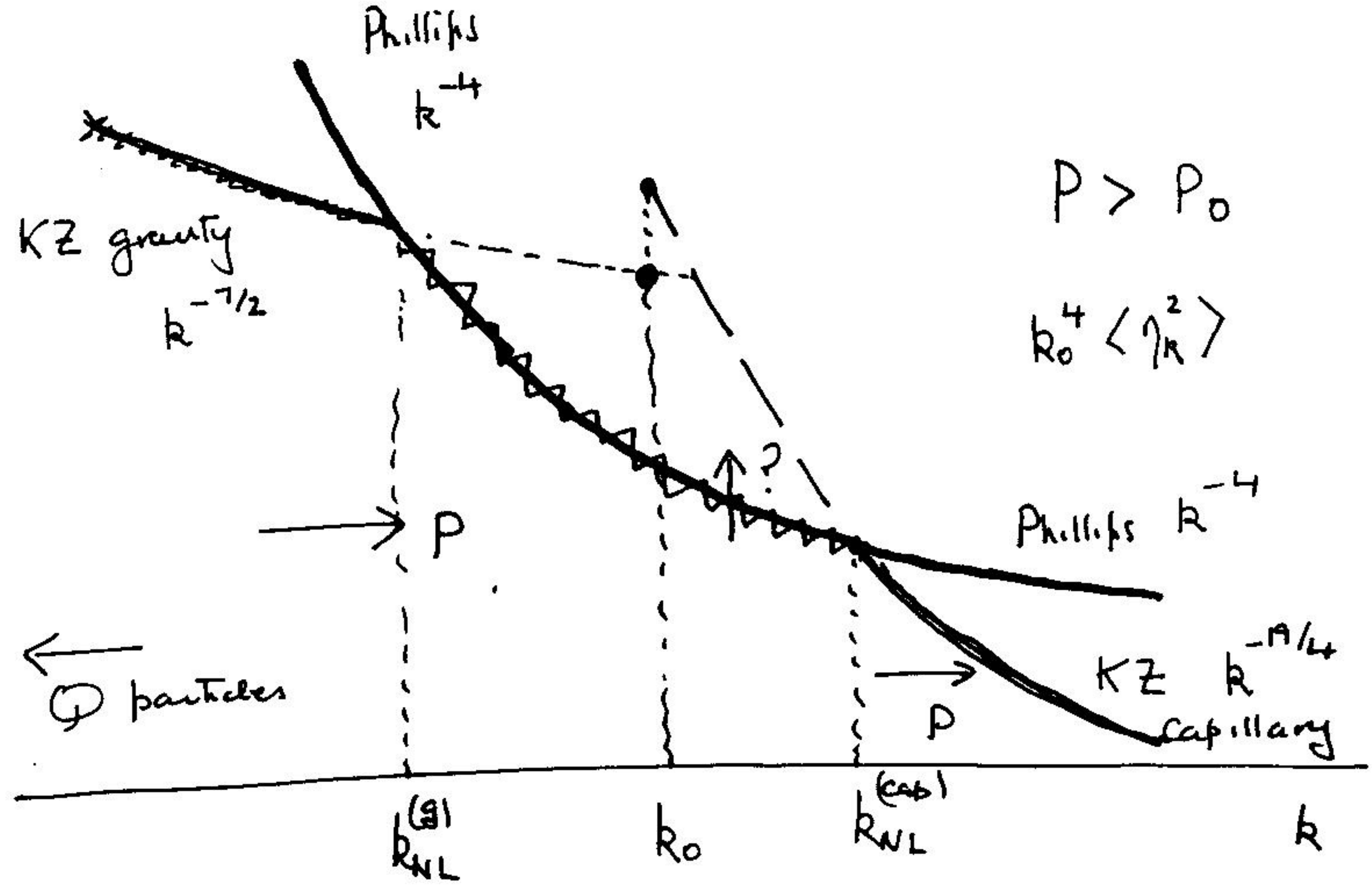
$$g^{-1} k_0^4 \, E_{\vec{k}}^{\text{Phillips}} = \left(\frac{k}{k_0}\right)^{-4}$$

$$\frac{t_L}{t_{NL}} = \left(\frac{P}{P_0}\right)^{2/3} \left(\frac{k}{k_0}\right) = \left(\frac{E_{\vec{k}}^{(g)}}{E_{\vec{k}}^{\text{Phillips}}}\right)^2$$

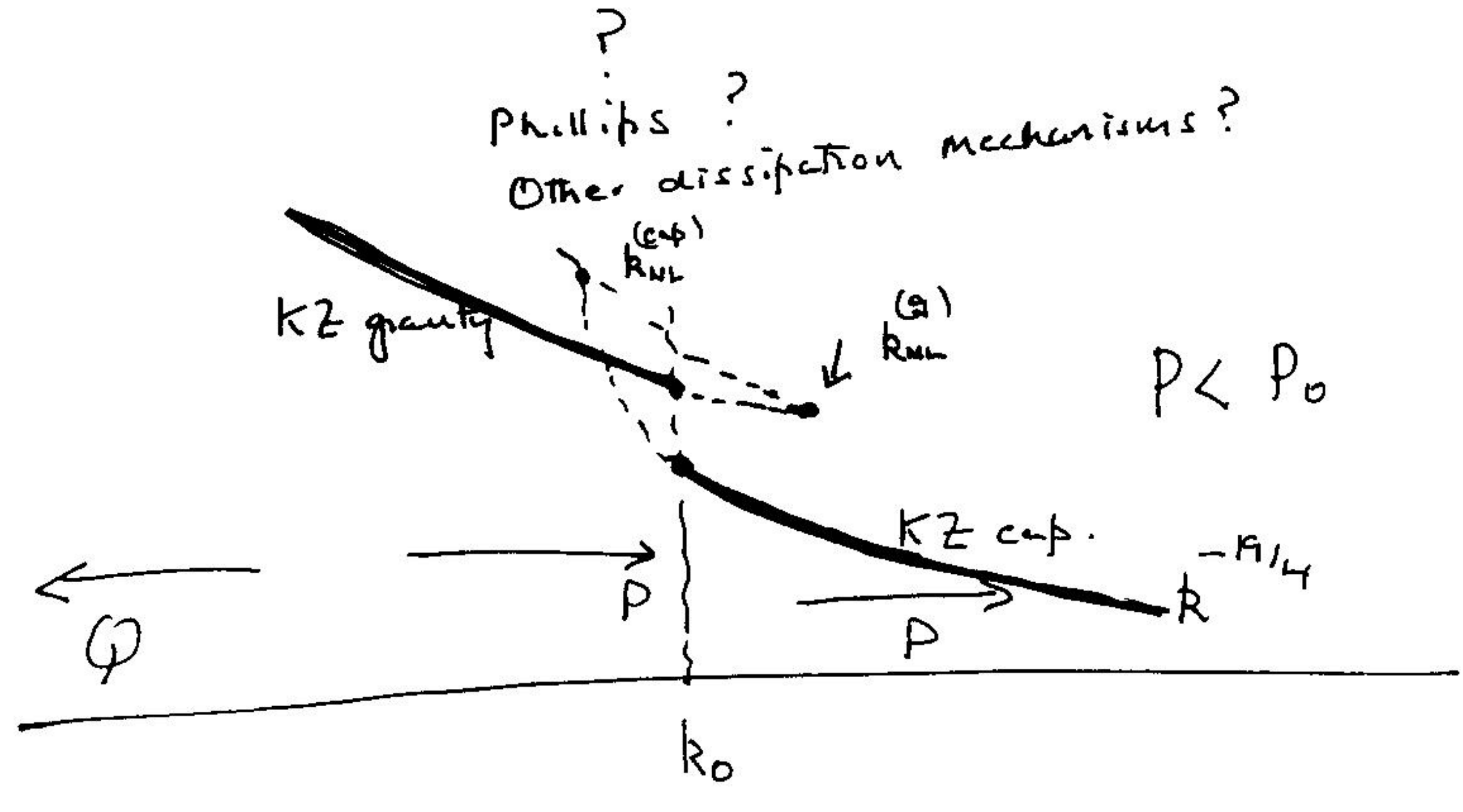
$$\sigma^{-1} k_0^2 \, E_{\vec{k}}^{(cap)} = C_2 \left(\frac{P}{P_0}\right)^{1/2} \left(\frac{k}{k_0}\right)^{-11/4}$$

$$\sigma^{-1} k_0^2 \, E_{\vec{k}}^{\text{Phillips}} = \left(\frac{k}{k_0}\right)^{-2} \Longrightarrow k_0^4 \langle \eta_k^2 \rangle = \left(\frac{k}{k_0}\right)^{-4}$$

$$\frac{t_L}{t_{NL}} = \left(\frac{P}{P_0}\right)^{\frac{1}{2}} \left(\frac{k}{k_0}\right)^{-3/4} = \frac{E_{\vec{k}}^{cap}}{E_{\vec{k}}^{\text{Phillips}}}$$



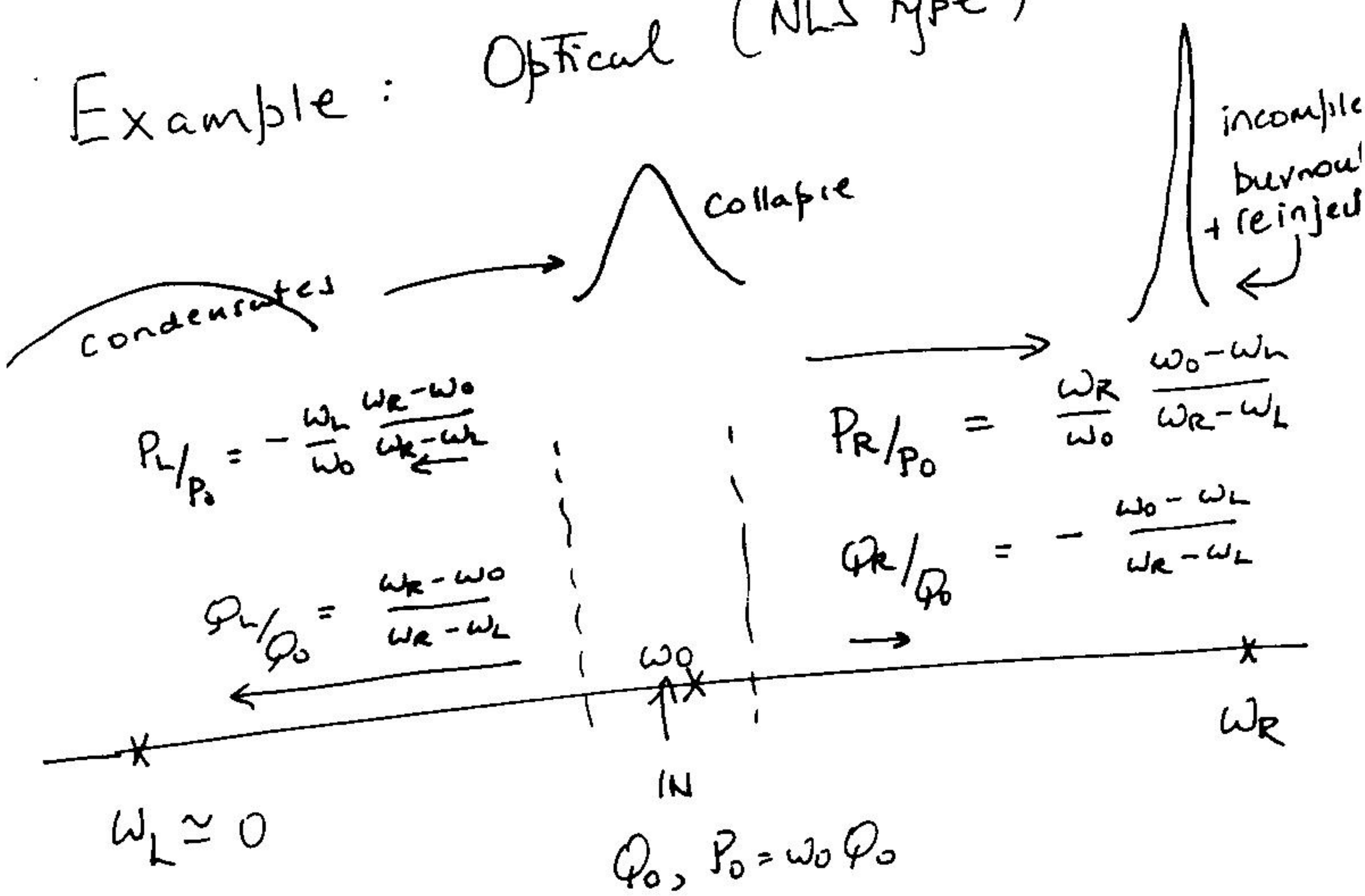
Phillips
k^{-4}
KZ gravity
$k^{-7/2}$
$P > P_0$
$k_0^4 \langle \eta_k^2 \rangle$
?
P
Phillips k^{-4}
φ particles
P
KZ $k^{-19/4}$
capillary
$k_{NL}^{(g)}$
k_0
$k_{NL}^{(cap)}$
k



?
Phillips ?
Other dissipation mechanisms?
$k_{NL}^{(cap)}$
KZ gravity
$k_{NL}^{(g)}$
$P < P_0$
KZ cap.
$k^{-19/4}$
P
P
φ
k_0

Why coherent structures / large fluctuations are inevitable in order to balance fluxes and allow the system to reach a statistically stationary state

Example: Optical (NLS type) turbulence



1. Particles and energy are inserted at intermediate frequencies ω_0.

2. Energy flows to $\omega = \infty$, particles to $\omega = 0$.

3. If no sink at $\omega = 0$ (e.g. laser), condensate builds.

4. Unstable condensate leads to collapsing filaments which each carry N_c mass + 0 energy to $\omega = \infty$.

5. Incomplete burnout fN_c leads to a reinjection of particles and energy to wave field.

6. Inverse flux builds up
$Q_0 + (1-f)Q_0 + (1-f)^2 Q_0 + \cdots \to \frac{1}{f} Q_0$
so that steady state can be achieved. SOLVABLE?

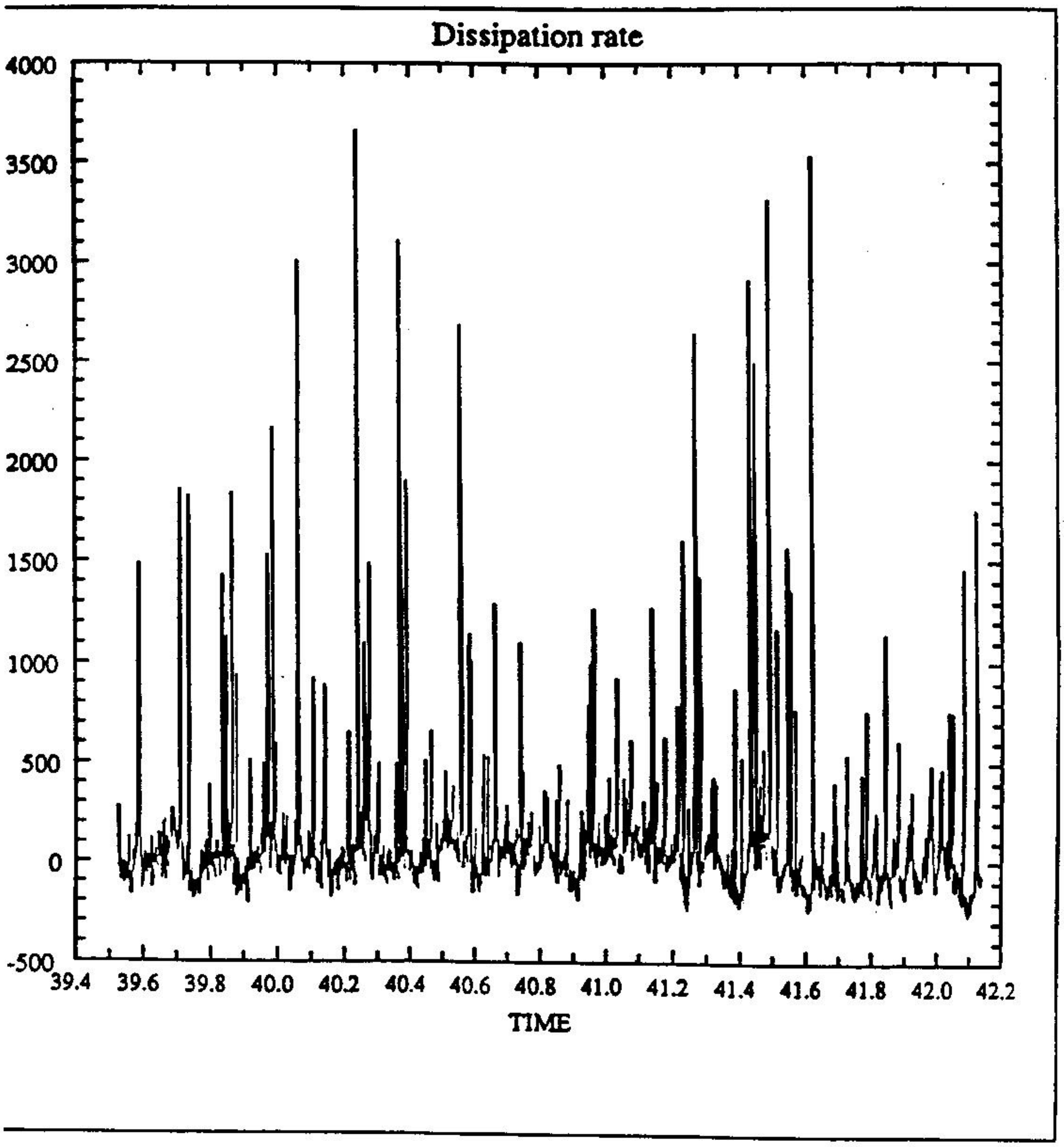

Dissipation rate
4000
3500
3000
2500
2000
1500
1000
500
0
-500
39.4
39.6
39.8
40.0
40.2
40.4
40.6
40.8
41.0
41.2
41.4
41.6
41.8
42.0
42.2
TIME

Figure 10