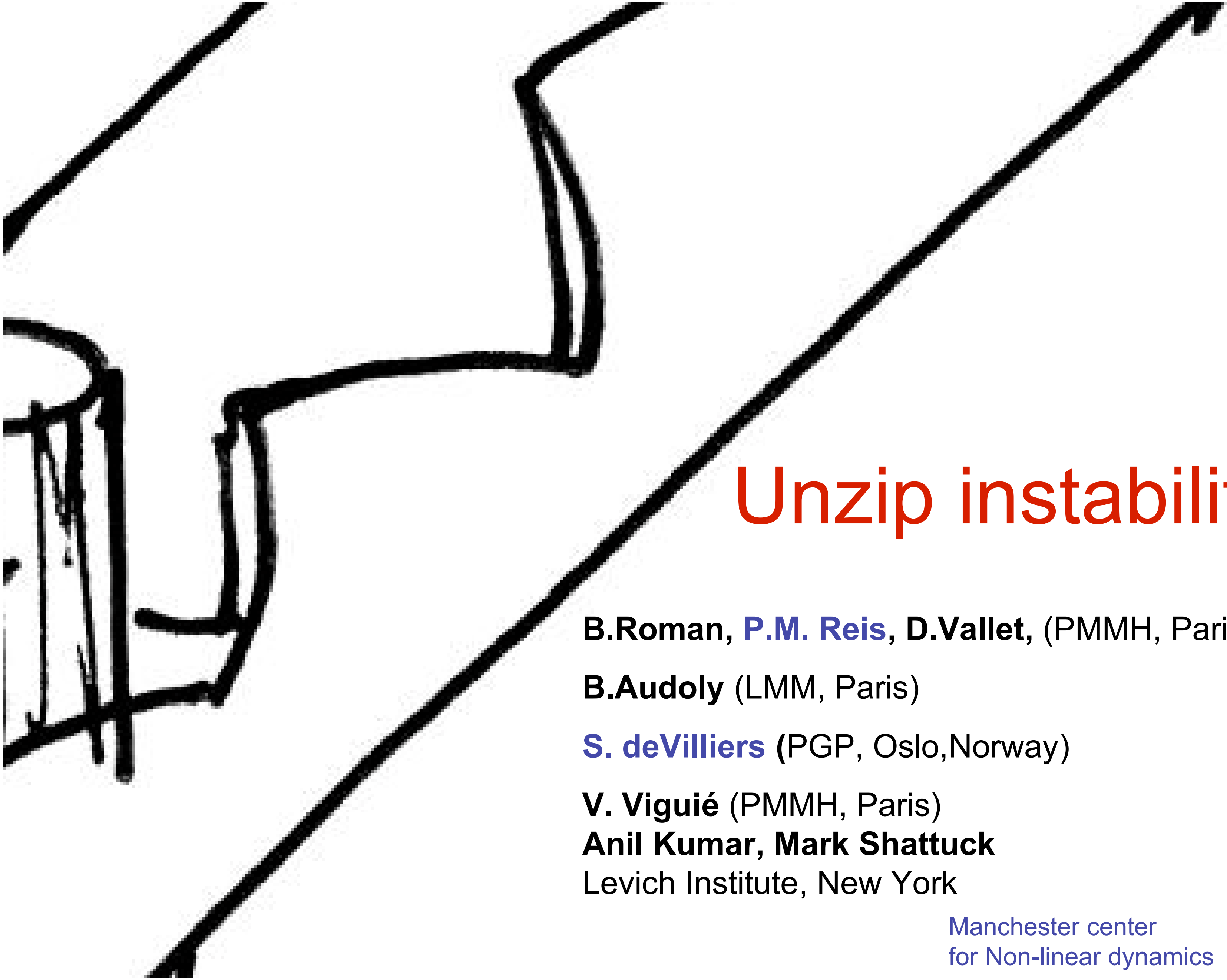


Unzip instability

B.Roman, P.M. Reis, D.Vallet, (PMMH, Paris)

B.Audoly (LMM, Paris)

S. deVilliers (PGP, Oslo,Norway)

V. Viguié (PMMH, Paris)
Anil Kumar, Mark Shattuck
Levich Institute, New York

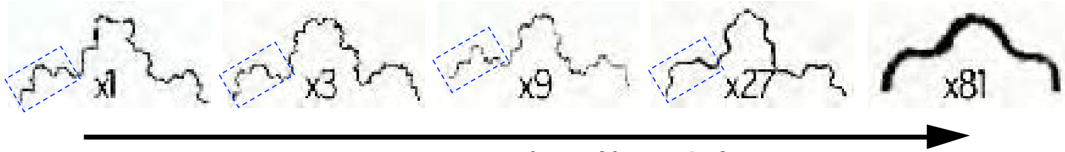
Manchester center
for Non-linear dynamics

I.4. Fracture of thin sheet with plastic deformation

If tear a thin plastic sheet...

... get cascade of oscillatory buckling

E. Sharon, B. Roman, M. Marder *et. al* Nature, 2002



Zoom experimental profile with factor x3

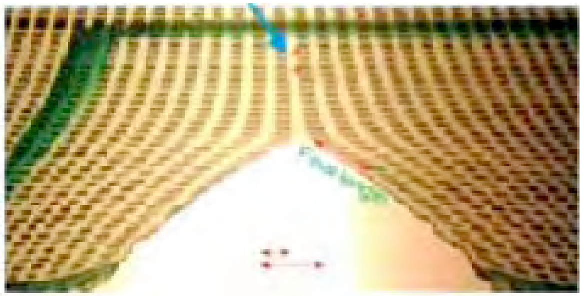


Photo: E. Sharon

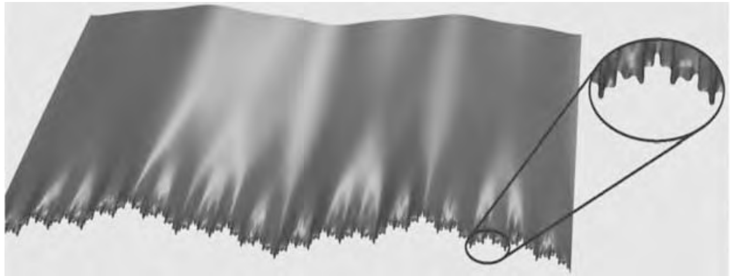
- Irreversible plastic deformation near crack tip;
- Change of the local metric near the edge of the crack lip;
- Minimum energy of the sheet achieved through bending:
  the sheet explores the 3rd dimension and buckles.

For remainder of our talk will use **brittle** thin sheets.
i.e. negligible plastic deformation near crack tip.

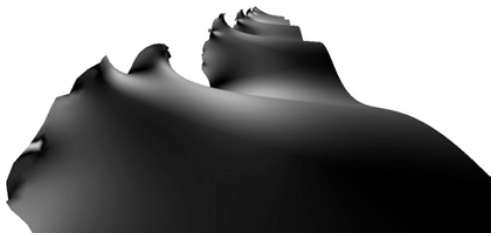
Torn plastic sheet:

E. Sharon, B. Roman, M. Marder *et. al* Nature, 2002

Thin sheet numerical solutions:



B. Audoly and A. Boudaoud PRL, 2003



M. Marder et. al. Europhys. Lett, 2003

Orchid flower:

Lettuce leaf:

Similarity between torn sheets and plant tissue!

I.1. Fracture. Why?

When something breaks,
can we know the shape of the resulting pieces?

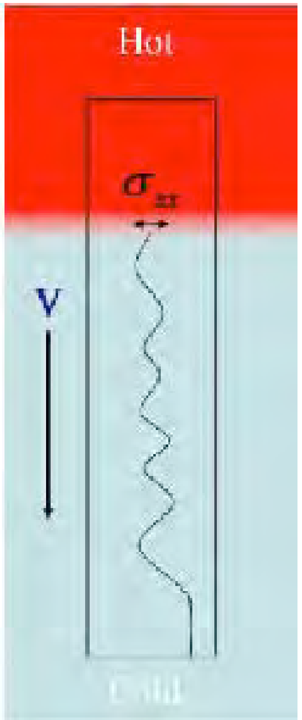
unadorned.org/dandruff/ archives/2002/03/

Can we predict the direction of propagation of the crack tip?

Recent upsurge in the study of oscillatory fracture.

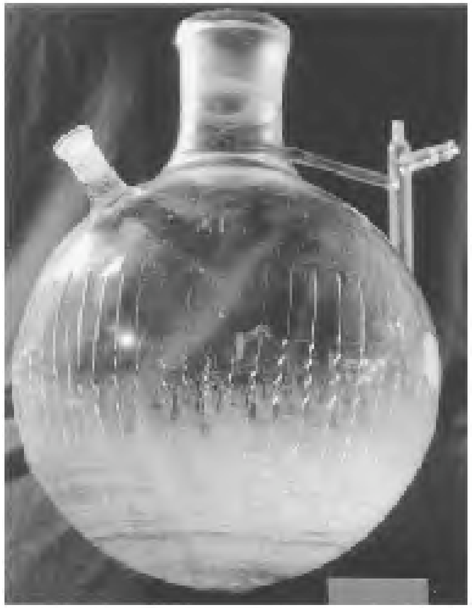
Problem still far from understood!

I.2. Quasi-static oscillatory fracture



Deegan 2003

- Thermally quench a thin strip
- Obtain oscillatory fracture paths
- Observed in strips of:
  - glass
  - silicon



Bahat 1991

Deegan et. al. 2003

Experiments:
- Yuse & Sano, 1993
- Ronsin et. al. 1995
- Yang & Ravi-Chandar, 2001
- Degan et. al, 2003

Theory:
- Adda Beddia & Pomeau, 1995
- Yang & Ravi-Chandar, 2001
- Bouchbinder et. al, 2003

It has recently been shown that
this instability emerges through a **Hopf bifurcation**.

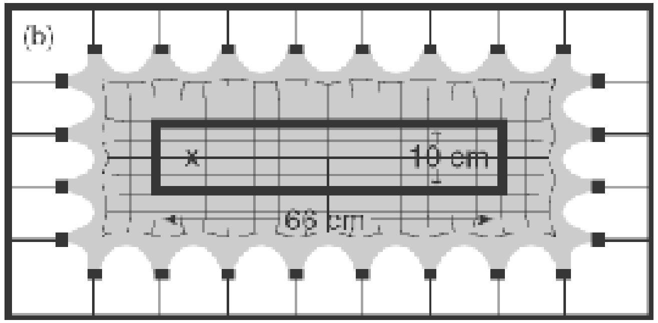
theory (Yang et. al.) and simulations (Bouchbinder et. al.)

I.3. Dynamic oscillatory fracture in rubber sheets

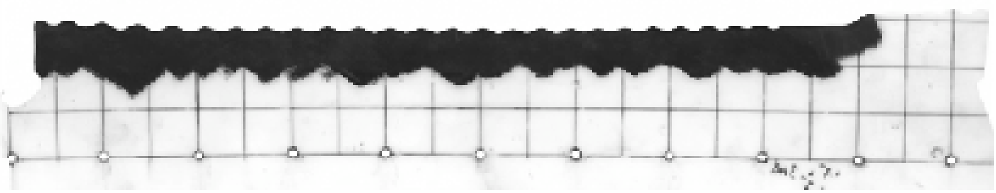
- Bi-axially tension a thin rubber sheet with typical strains of $\varepsilon_{x,y}$=1.0-2.5.
- Pierce sheet to initiate dynamic crack.
- Can obtain oscillatory crack paths.

Experiments:
- Deegan et. al., 2003

Theory:
- Henry & Levine, 2004



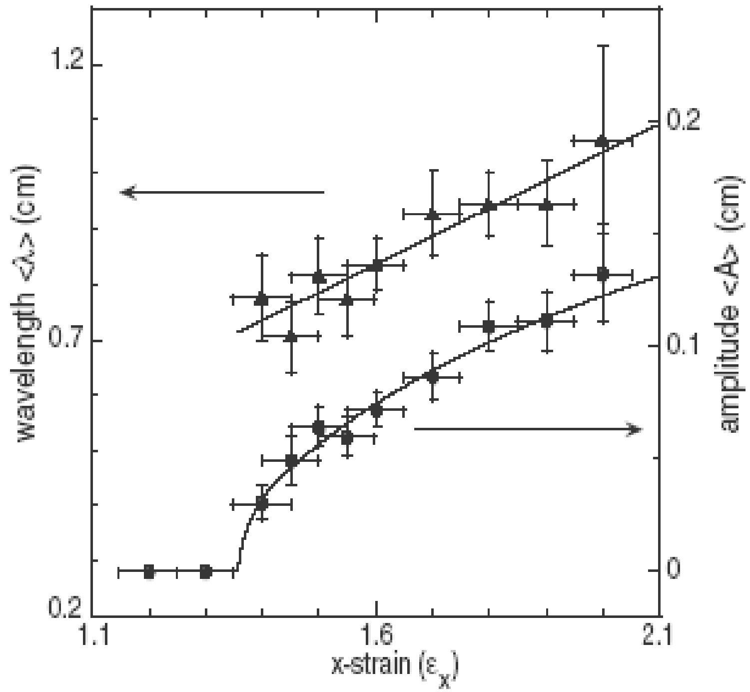
Deegan et. al., 2003



Deegan et. al., 2003

Suggested that this instability is a **Hopf Bifurcation:**

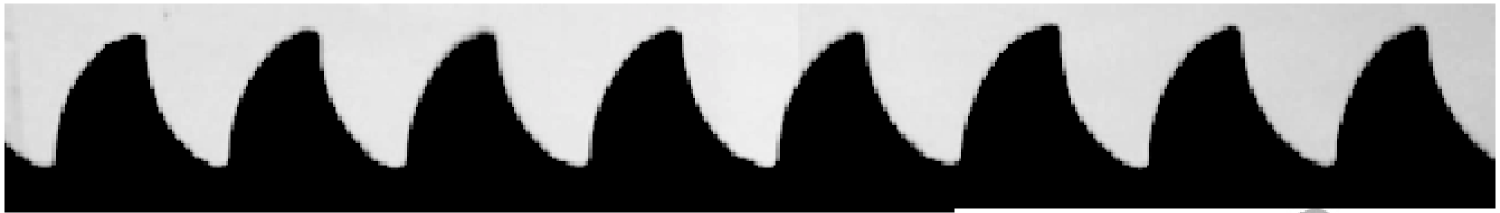
1) Square-root scaling near bifurcation point.
2) Close to the transition point, wavelength has finite value.



Deegan et. al., 2003

A robust periodic path…

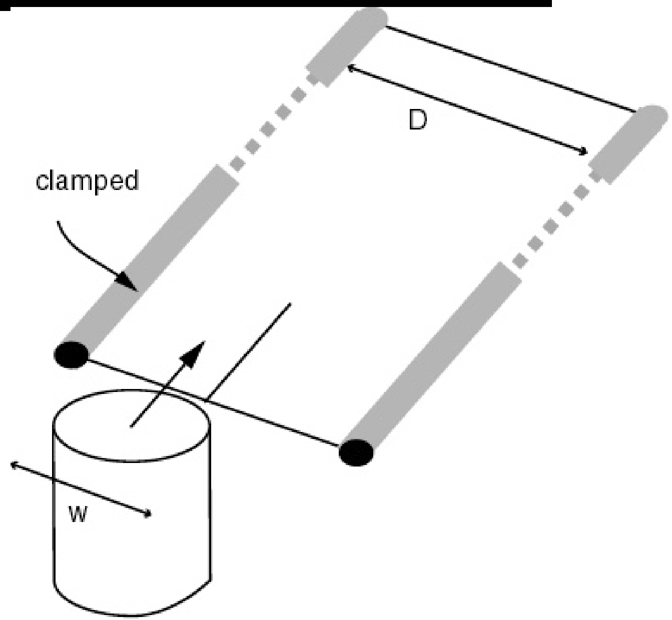
Also observed simultaneously by Ghattak & Mahadevan



Cut a thin sheet (thickness h, size D)

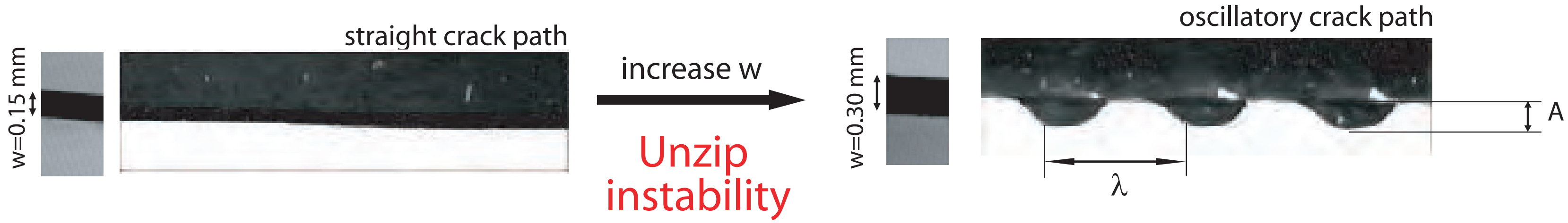
with a blunt object (width w)

at speed v

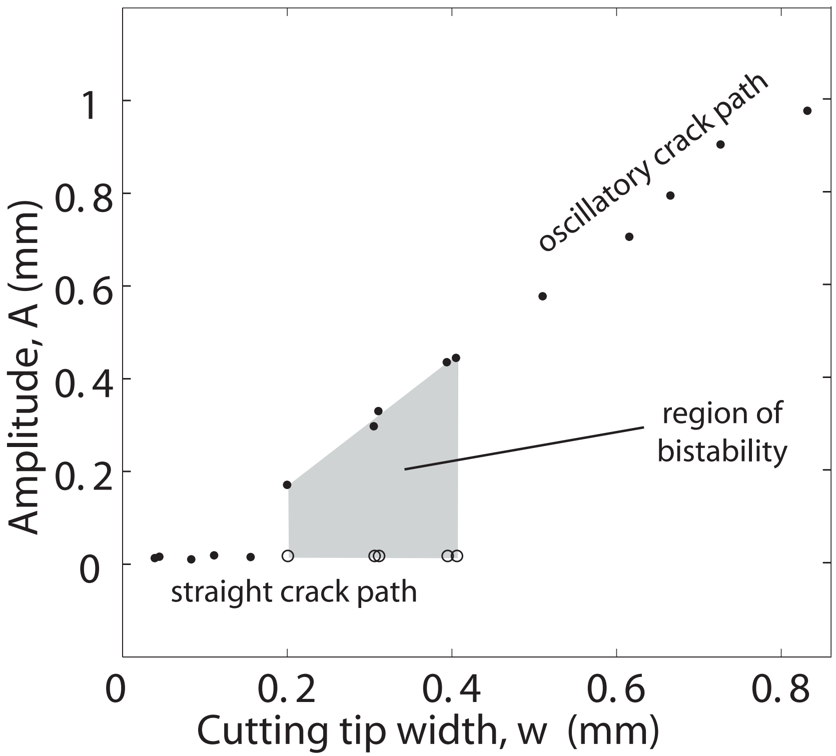


Bi-Oriented Polypropylene
or cellulose acetate

II.4 The Unzip instability



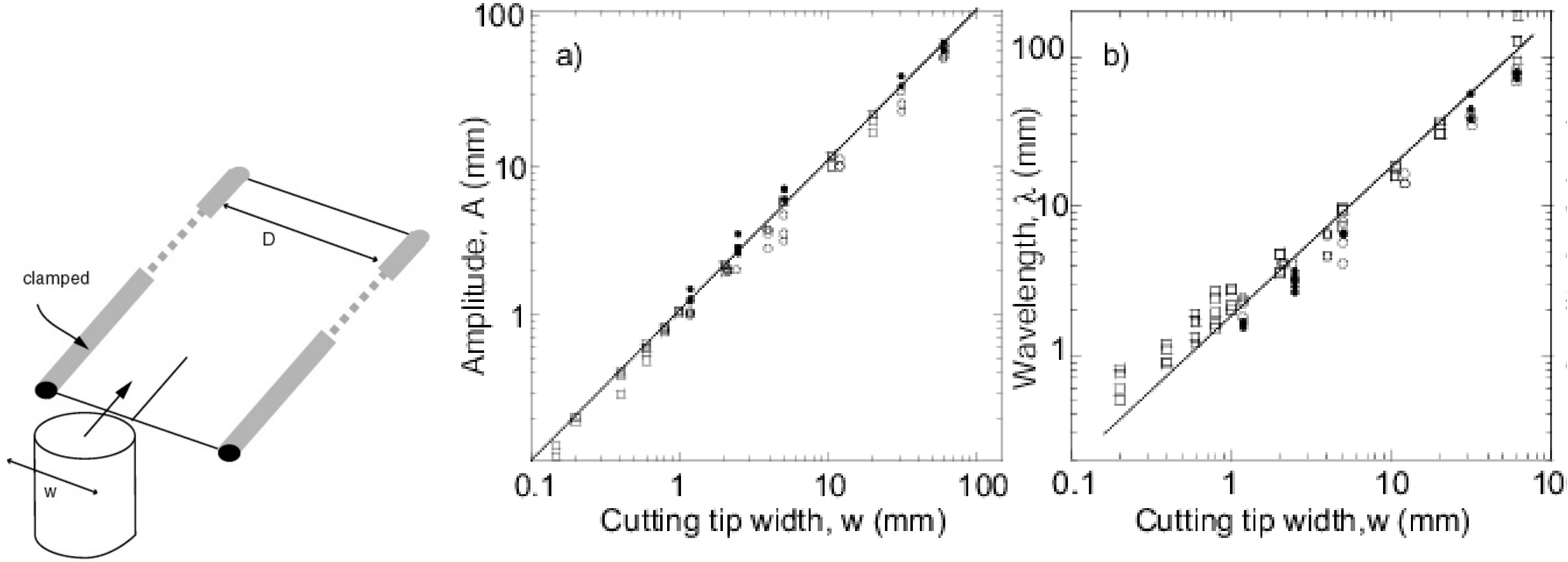
Transition diagram:



- Transition as a function of size of cutting tool
- Threshold for width of cutting tool is $w_c = 0.2$mm
- Bistability near w_c:
  - Both oscillatory and straight cracks can be observed.
  - Also observed in the thermal quenching experiments with silicon strips (Deegan, 2003).
- Linear dependence of A on w above threshold.

Far above theshold

Pattern is indépendant
- of cutting speed v
- of thickness h and material
- of distance D to B.C.

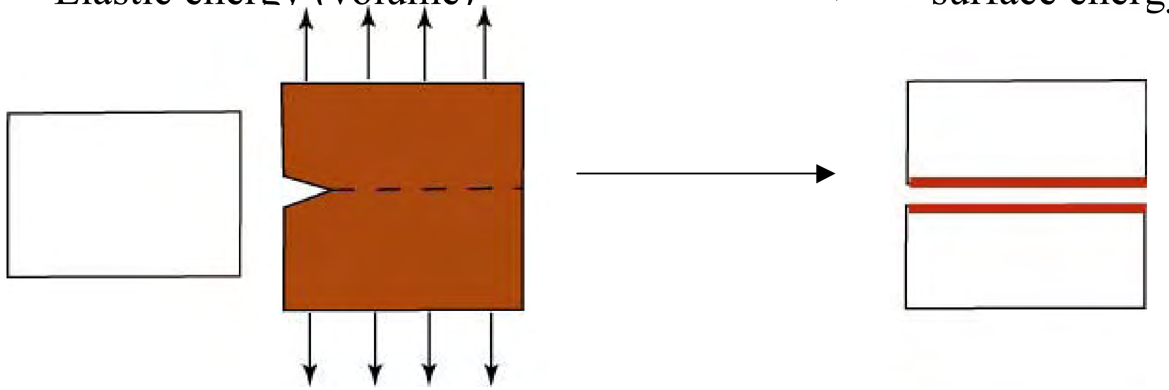


Amplitude, wavelength are linear with object width w

Fracture in thin sheets

crack : singular object

Elastic energy (volume) → surface energy



Thin sheet thickness h :

Stretching energy $E_e \sim h$

Bending energy $E_c \sim h^3$

Model : bending $E_c = 0$.

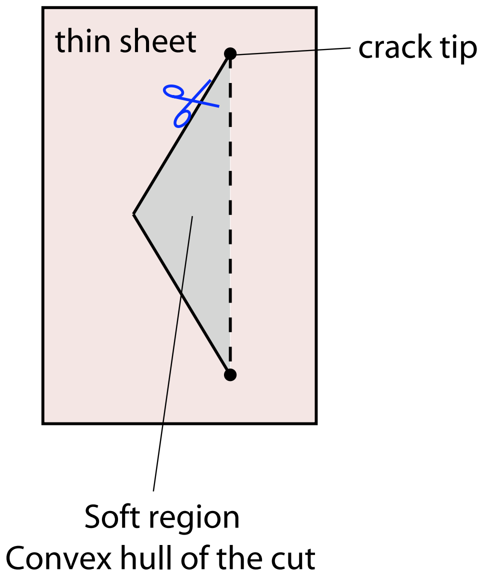
A set of *geometrical* rules for propagation

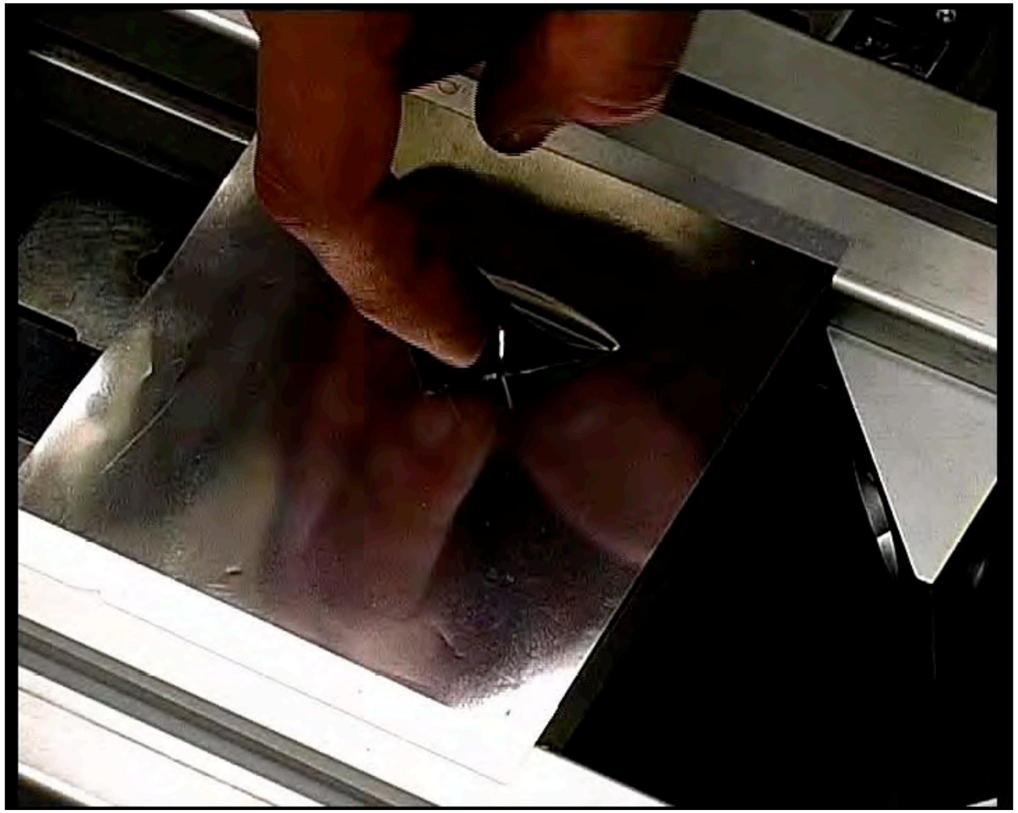
III.1 Towards a geometrical model - ingredients (a)

- Main properties of the **thin** sheets we are using:

  1. Brittle:
     (negligible plastic deformations);
  2. Low bending modulus:
     (negligible bending energy).

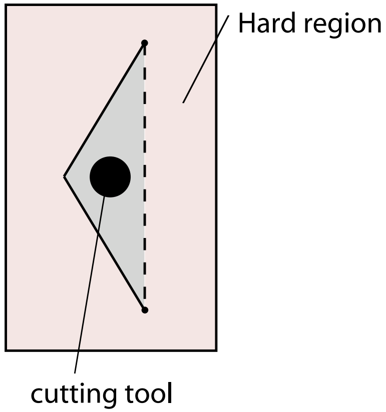
- Consider a triangular cut in the sheet:



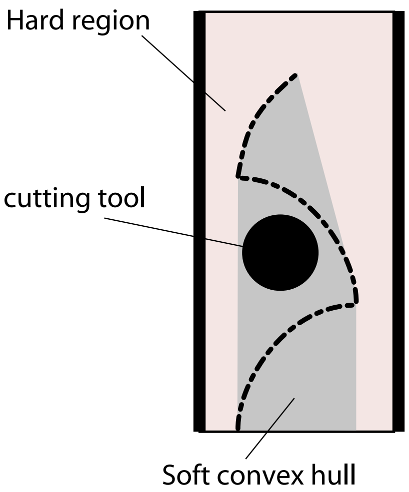


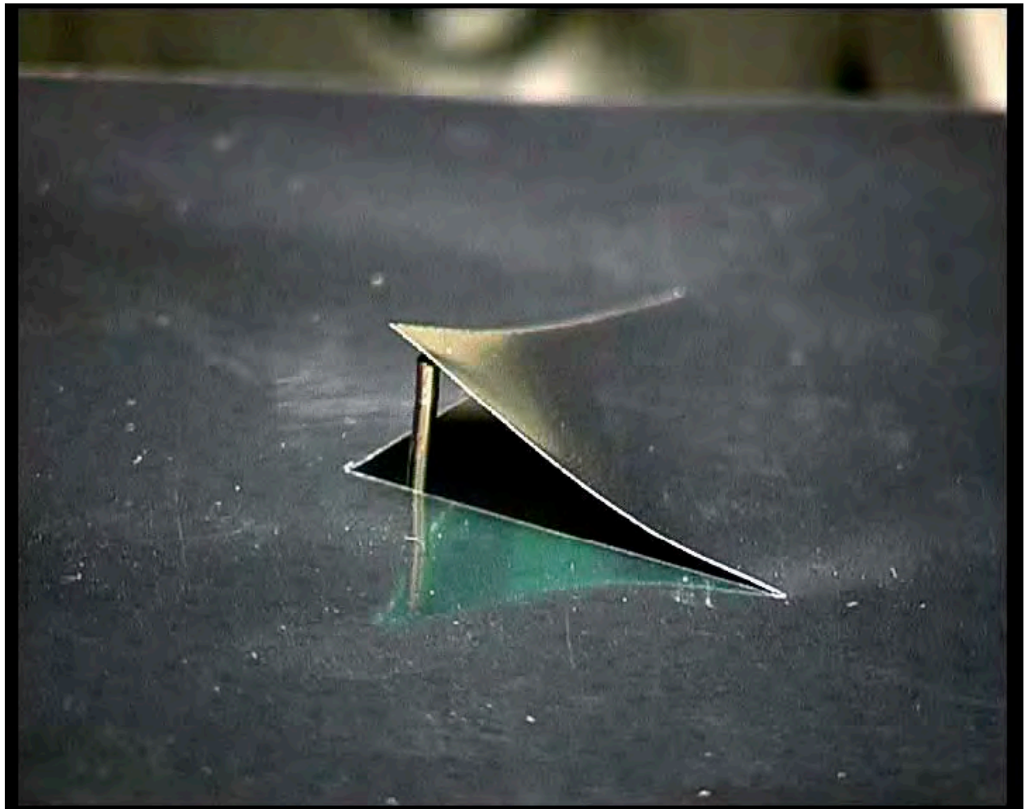
III.1 Towards a geometrical model - ingredients (b)

- Insert cutting tool within soft region:



- 2D projection of geometry of experiments:

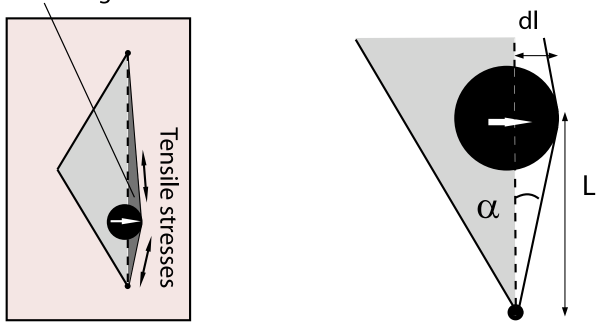




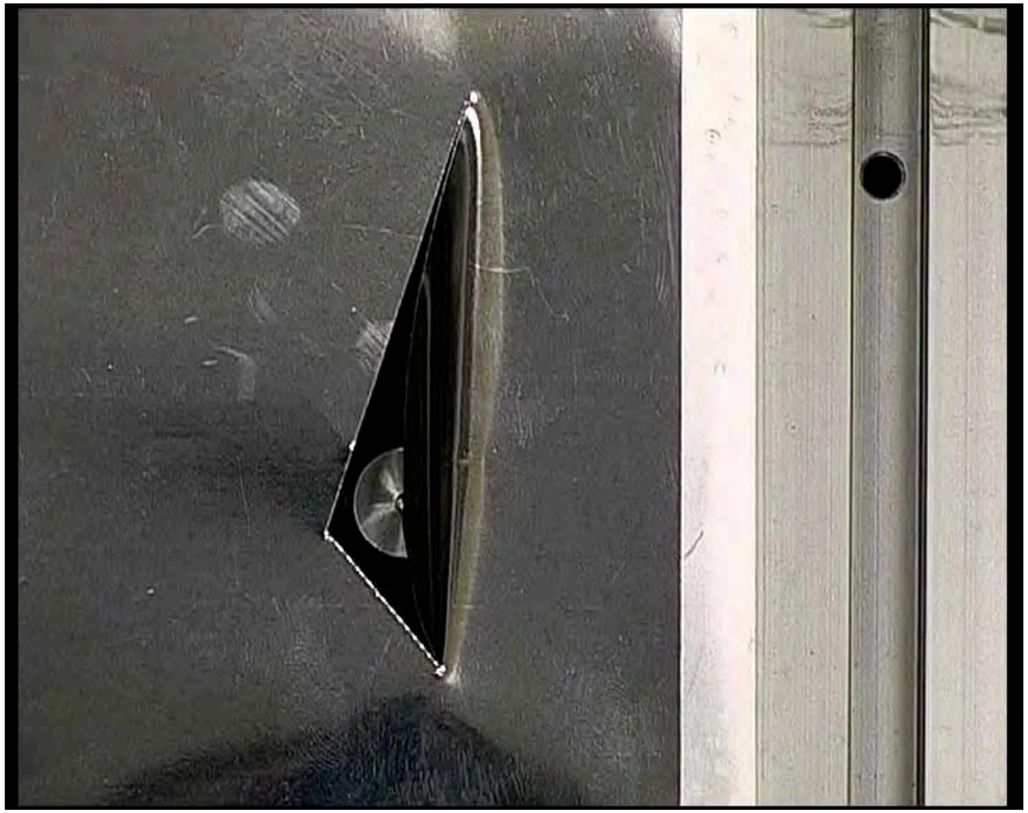
Cutting tool can be displaced anywhere within soft zone with negligible stretching (out-of-plane bending only).

III.1 Towards a geometrical model - ingredients (c)

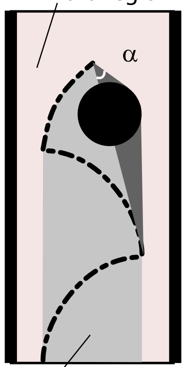
- Displace cutting tool into hard region.



=> This leads to in-plane stretching.



- Energy stored in active region: $\xi = Ye^2 v$



- v is volume of strained material
- Y is Young's modulus
- Strain:

$$e = \frac{\frac{L}{\cos \alpha} - L}{L} \sim \alpha^2 \quad \text{for } \alpha \ll 1$$

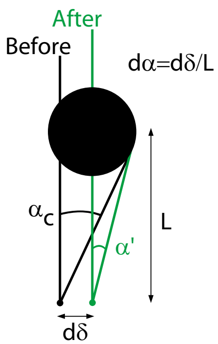
$$\xi \sim Y\alpha^5 L^2 h$$

III.1 Towards a geometrical model - ingredients (d)

- Griffith's criterion:

- Imagine crack advances by $d\delta$:

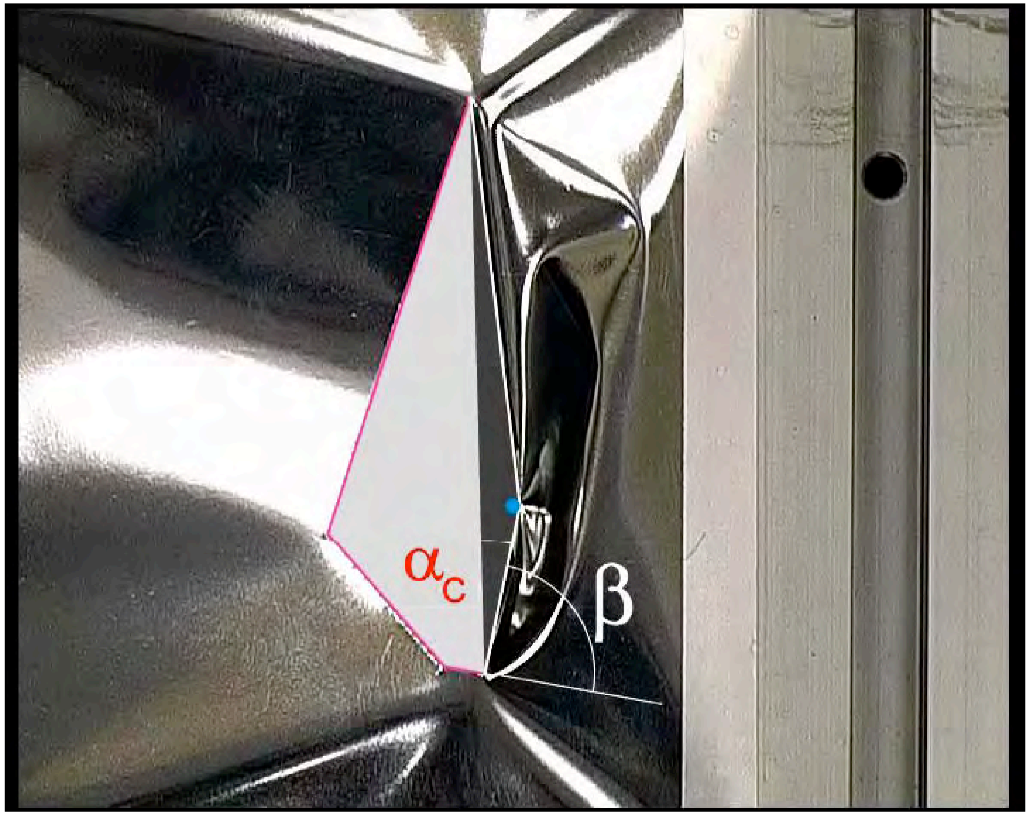
  => **Release of elastic energy**



For propagation:

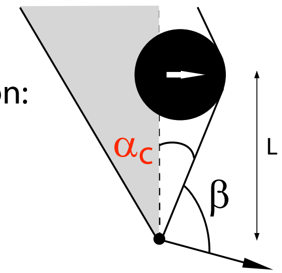
Δ (Elastic energy) $\geqslant$ Fracture energy

$$\underbrace{Y\alpha^4 L^2 h d\alpha}_{d\xi} \geqslant \underbrace{\Gamma h L d\alpha}_{G.ds}$$



- Critical angle for propagation:

$$\Rightarrow \alpha_c = \left[\frac{\Gamma}{YL}\right]^{1/4}$$



Hence, have 2 physical parameters:

1. Critical Angle for Propagation α_c. (from Griffith's criterion)
2. Propagation along direction β. (from principle of local symmetry)

II.4 Experimental scaling of α_c

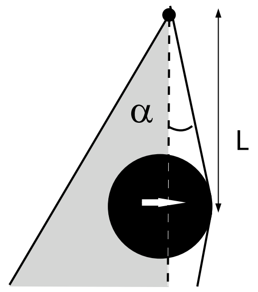
- Order of magnitute estimation of α_c :

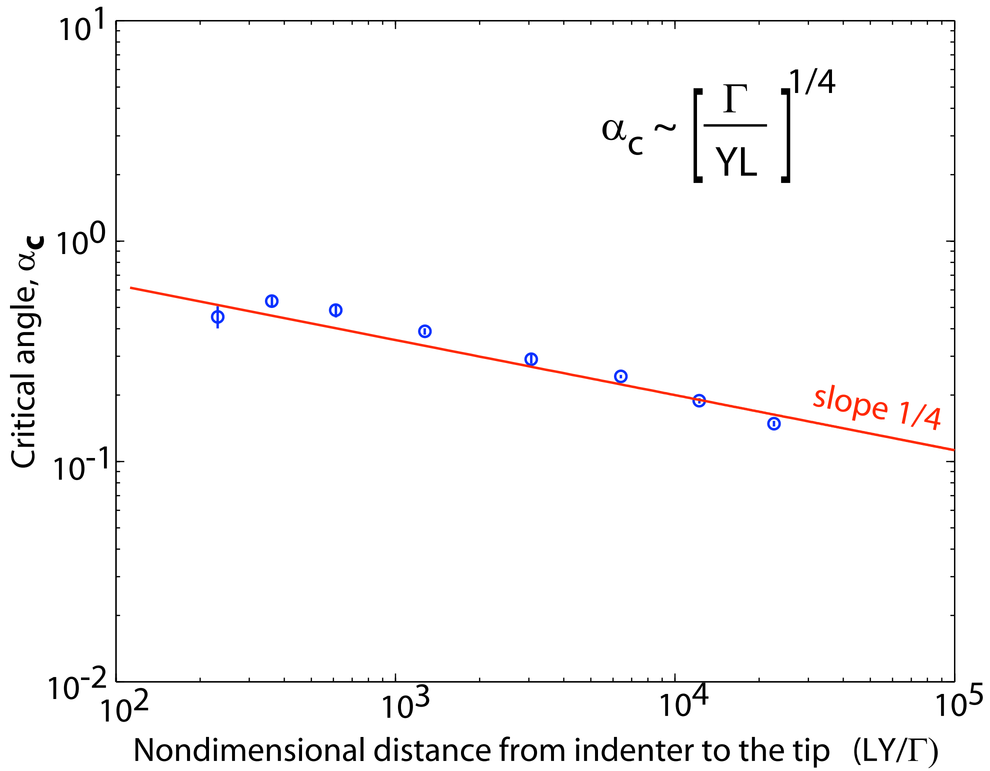
$\Gamma \sim 10^3$ N/m
$Y \sim 10^9$ Nm^2
$L \sim 10^{-2}$ m

→ $a_c \sim 10^{-1}$ rad

in experiments $0.1 < a_c^{exp} < 0.4$ ✓

- Scaling of α_c with lengthscale L (distance to the cutting tool):

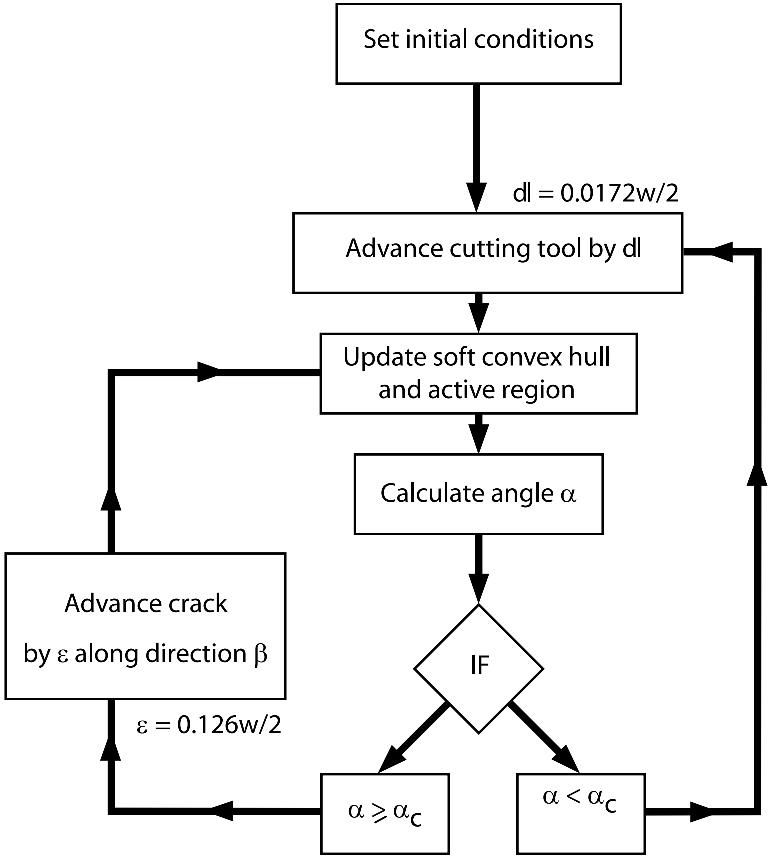


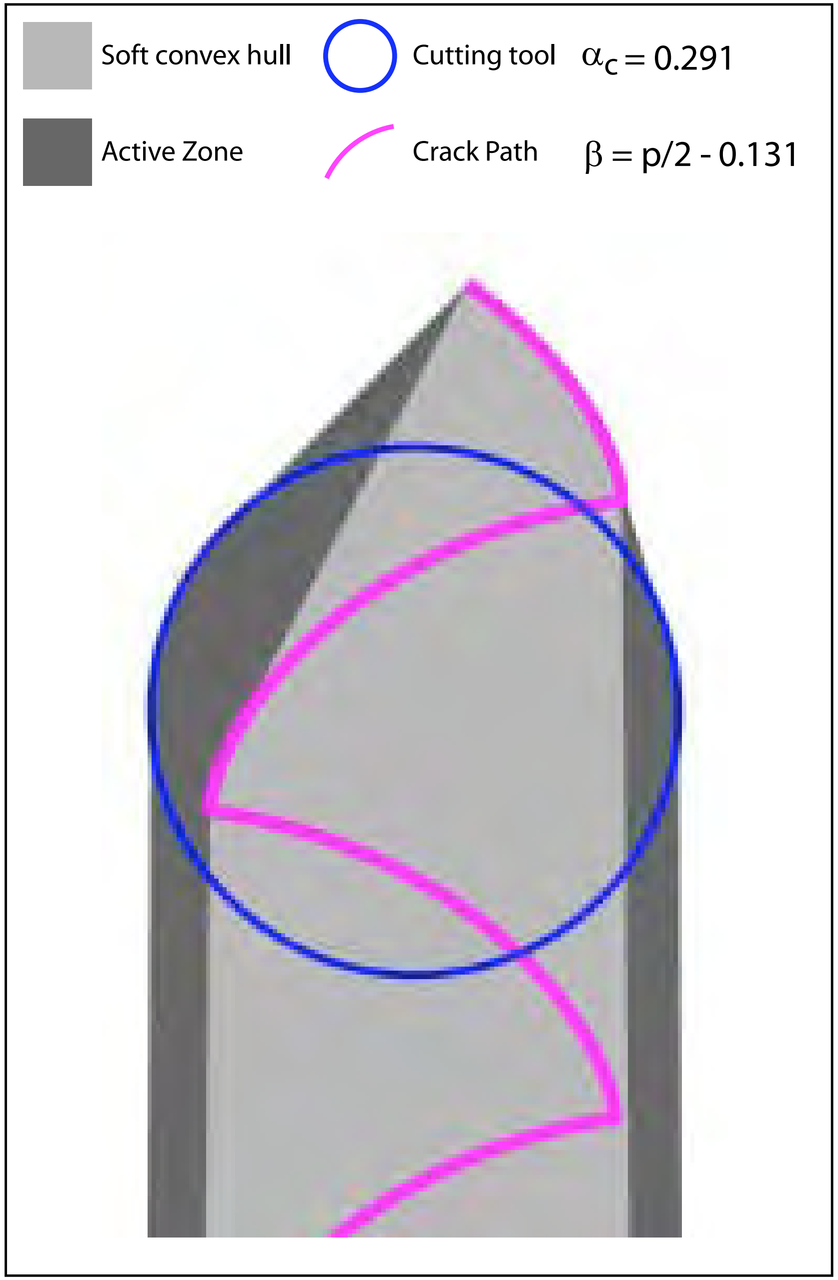


Scaling of α_c is consistent with our formulation.

III.2 Numerical Simulations

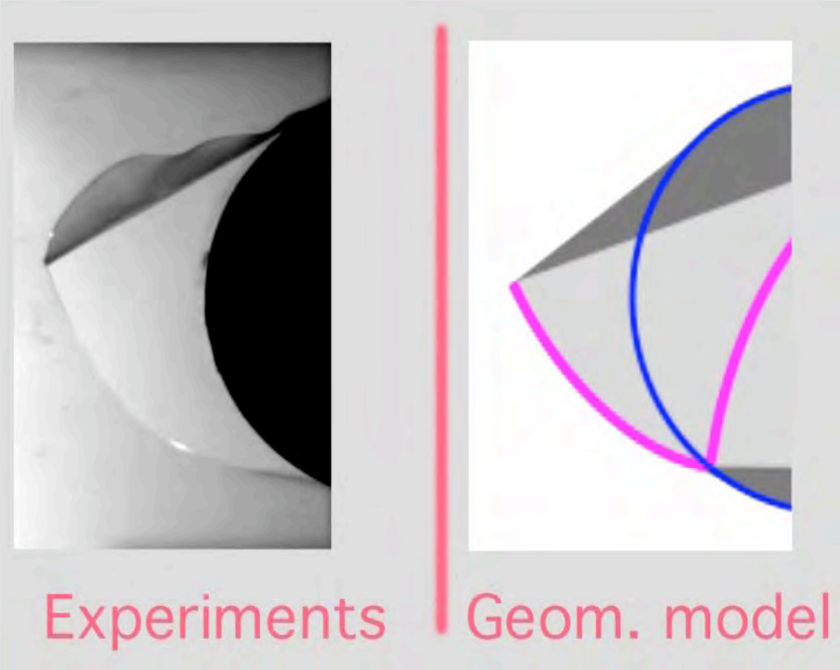
Geometrical model for fracture in thin films





Note: $\varepsilon >> dl$
i.e. separation of scales between dynamic and quasi-static regimes

IV. Comparison: Experiments & Model

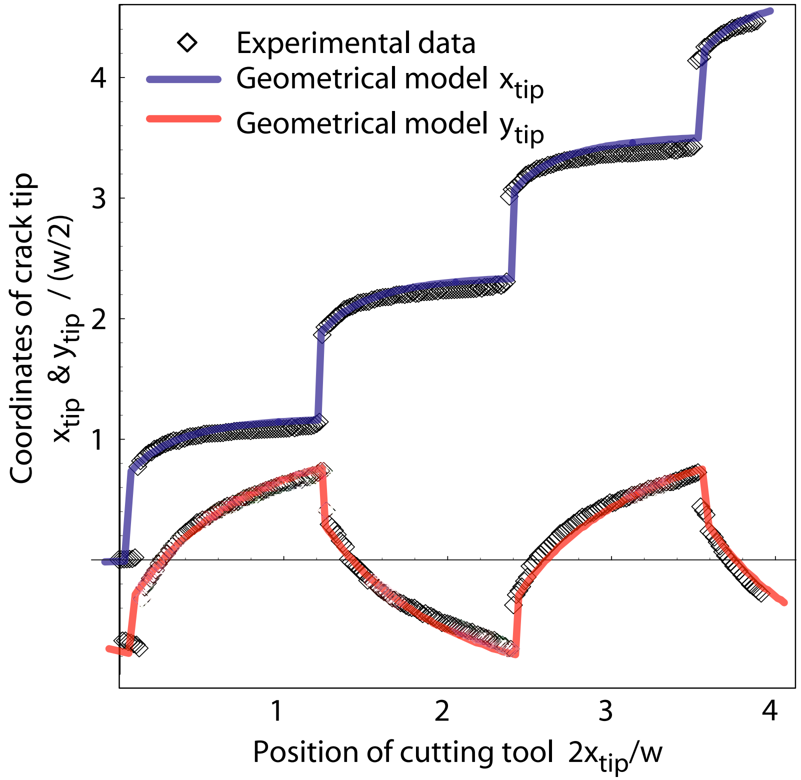


w = 31 mm
v = 1.3 mms^{-1}
D = 100 mm

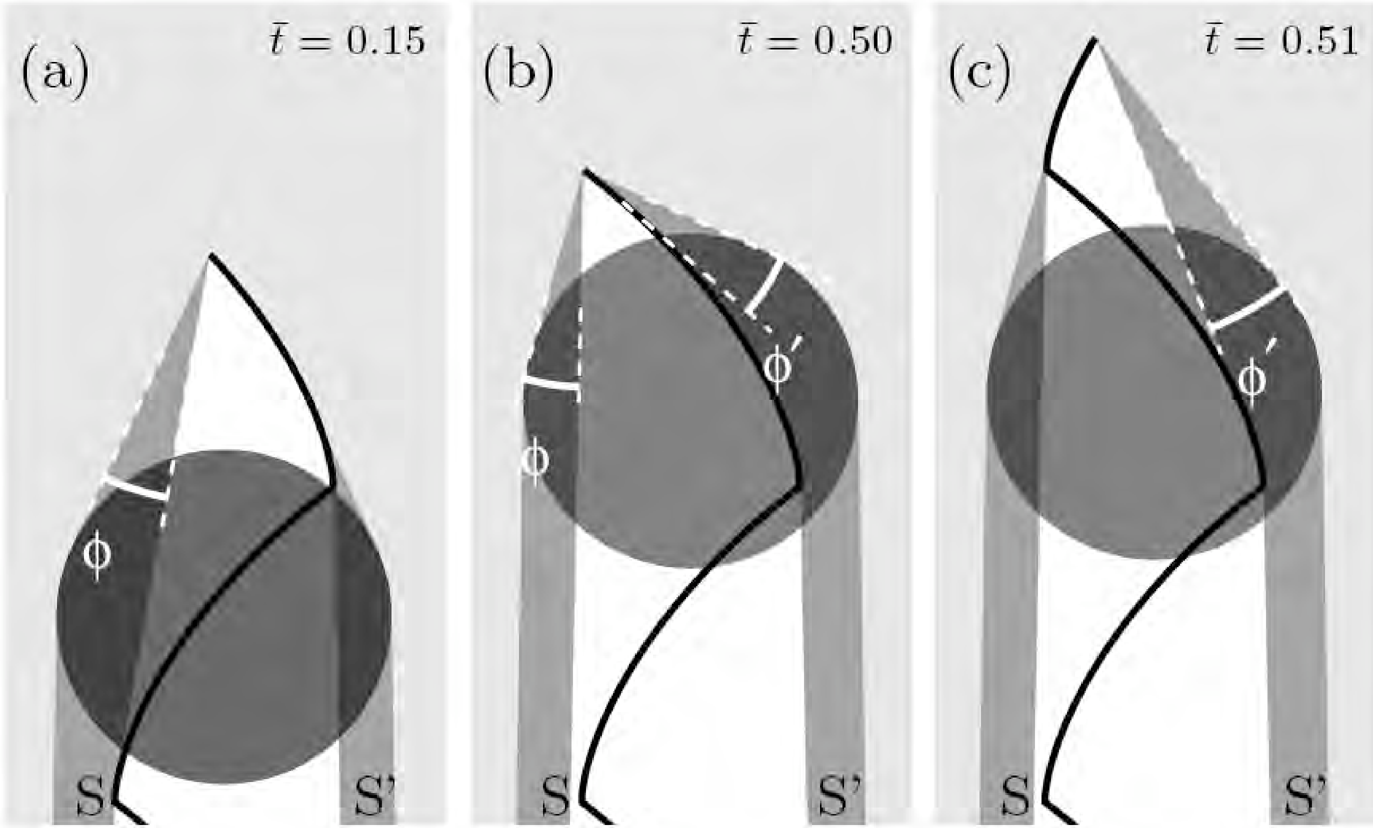
$\alpha_c = 0.291$
$\beta = \pi/2 - 0.131$

Parameters α_c and β were tuned for:

1. matching of exp and model periods;
2. under this constrains, best fit;



Excellent agreement between experiments and geometrical model for oscillatory fracture, far from threshold!



Kinks and dynamic burst are intrinsic to the geometrical rules:

due to sudden release of energy stored in S' region

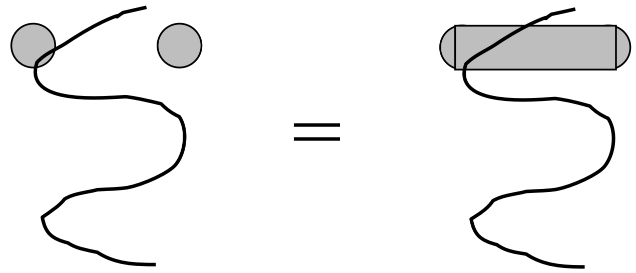
Différentes formes de l’indenteur ?

Un double indenteur

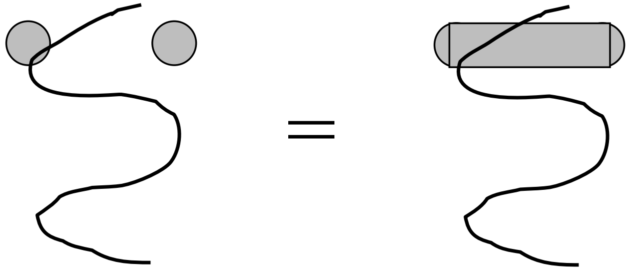
Différentes formes de l'indenteur ?

Un double indenteur

C'est l'enveloppe convexe de l'indenteur qui compte

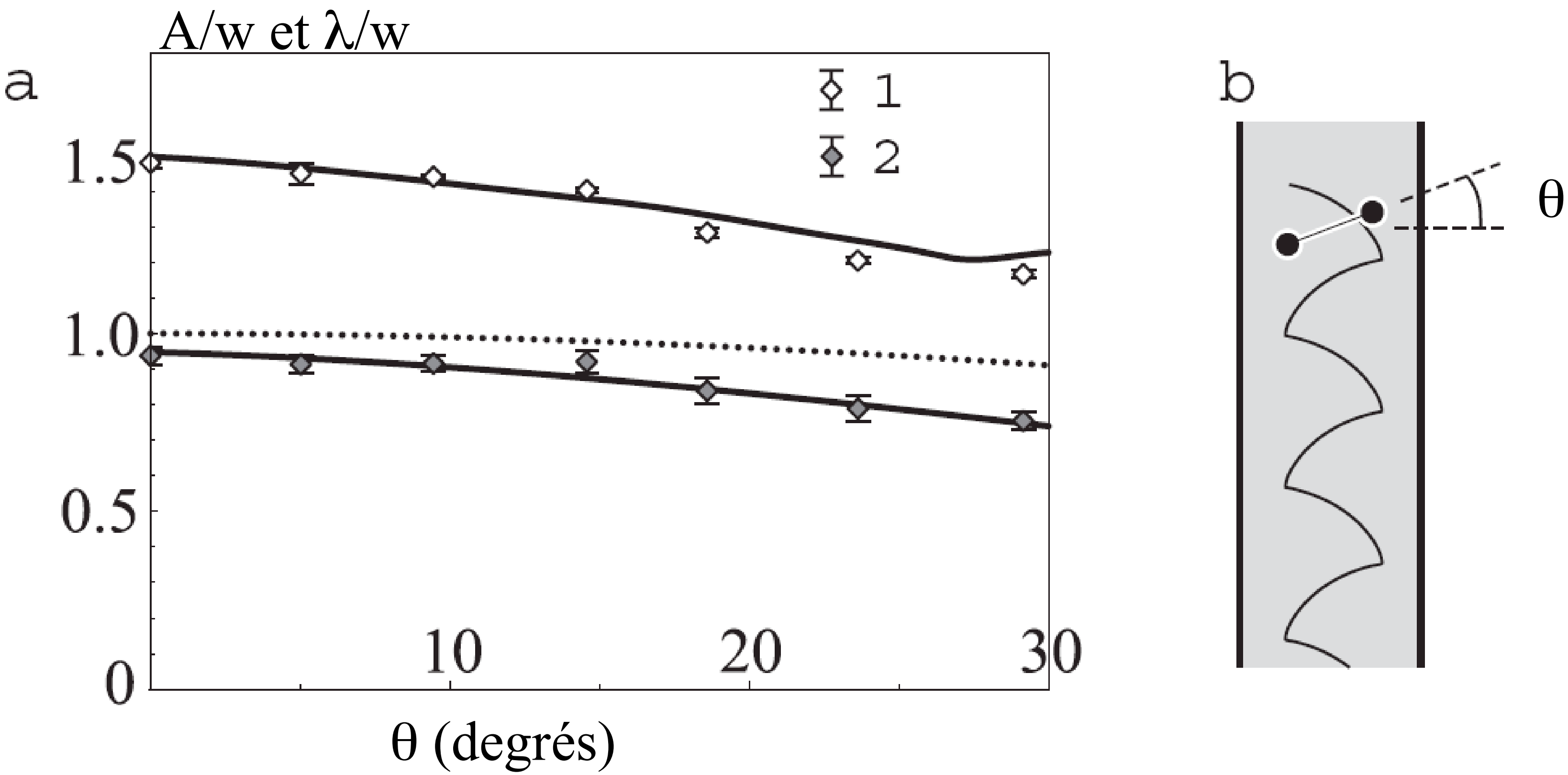


Différentes formes de l'indenteur ?



Un double indenteur

C'est l'enveloppe convexe de l'indenteur qui compte



Wake instability :

• a new fracture-path instabilty

• simplified crack+thin sheet theory

• cracks obey *geometrical* propagation rules

•Stress localized around object : no effect of BC.

REFERENCES :

B.Roman B.Audoly PM Reis S de Villiers **C.R.Acad.S**. 3331 (2003)

A.Gathak and L.Mahadevan **PRL** 2003

B.Audoly PM Reis, B.Roman **PRL** 2005